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CORRELATION MEASUREMENTS IN THE
BOUNDARY LAYER OF A FLAT PLATE

WILLIAM L. MARTIN III
AND
ARTHUR T. WHITE

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CORRELATION MEASUREMENTS IN THE
BOUNDARY LAYER OF A FLAT PLATE

by

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Submitted in Partial Fulfillment
of the Requirements for the
Degree of Master of Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

July, 1958

Cambridge, Massachusetts
July 21, 1958

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the requirements of the degree
of Master of Science, we herewith submit a thesis entitled
"Correlation Measurements in the Boundary Layer of a Flat
Plate."

ABSTRACT

CORRELATION MEASUREMENTS IN THE
BOUNDARY LAYER OF A FLAT PLATE

by

WILLIAM L. MARTIN, III, LIEUTENANT, U. S. NAVY

ARTHUR T. WHITE, LIEUTENANT COMMANDER, U. S. NAVY

Submitted to the Department of Nuclear Engineering
on July 21, 1958, in partial fulfillment of the require-
ments for the degree of Master of Science.

The objective of this thesis was to assemble an instrumentation system and make limited measurements of correlation quantities in the boundary layer of a flat plate with an unheated starting length.

A system consisting of suitable probes, amplifier, calibration and measuring circuits, and voltmeter was assembled and tested. A series of measurements of the characteristic quantities of turbulent fluid flow were made by a traverse through the boundary layer at one location on the plate.

Although there is considerable scatter in the results, the system can be expected to give satisfactory results after the recommended equipment modifications and test procedures are effected.

Thesis Supervisor: R. J. Nickerson
Title: Assistant Professor of Mechanical Engineering

ABSTRACT

CONVERSION MECHANISM IN THE

ACQUANT LAYER OF A FLAT PLATE

by

WILLIAM T. MARTIN, III, LIEUTENANT, U. S. NAVY

WILLIAM T. MARTIN, LIEUTENANT COMMANDER, U. S. NAVY

Submitted to the Department of Physics, Massachusetts Institute of Technology, in partial fulfillment of the requirements for the degree of Doctor of Science.

The objective of this thesis was to establish an instrumentation system and make limited measurements of conversion quantities in the boundary layer of a flat plate with an upstream acoustic input.

A system consisting of multiple probes, amplifier, calibration and measuring circuit, was designed and assembled and tested. A series of measurements of the conversion quantities of turbulent flow was made by a traverse through the boundary layer at one location on the plate.

Although there is considerable scatter in the results, the system was found to give satisfactory results after the recommended equipment modifications and test procedures are followed.

Thesis Supervisor: R. J. Scheraga
Assistant Professor of Mechanical Engineering
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1. INTRODUCTION

In the theory of turbulent fluid flow and heat transfer, the correlated quantities $\overline{uv}$ and $\overline{vt}$ have particular significance. The simplified equations that describe the boundary layer over a heated flat plate may be written as: (See Appendix B for derivation)

Momentum transfer:

$$\frac{\partial}{\partial x} (\bar{P} + \rho \bar{u}^2 + \rho \bar{u}^2) = \frac{\partial}{\partial y} (\mu \frac{\partial \bar{u}}{\partial y} - \rho \overline{uv} - \rho \overline{uv}); \quad (1)$$

Energy transfer:

$$\frac{\partial}{\partial x} (\bar{uT} + \bar{ut}) = \frac{\partial}{\partial y} (k \frac{\partial \bar{T}}{\partial y} - \overline{vT} - \overline{vt}). \quad (2)$$

Analytical solution of these equations is possible only under very restrictive conditions. For such solutions the following definitions of the eddy shear stress coefficient, ϵ_m and the eddy energy transfer coefficient, ϵ_h are introduced:

$$\overline{uv} = -\epsilon_m \frac{\partial \bar{u}}{\partial y} \quad (3)$$

and

$$\overline{vt} = -\epsilon_h \frac{\partial \bar{T}}{\partial y} \quad (4)$$

Assumed analogies between these coefficients together with suitable boundary conditions has been an accepted method for solving problems for a great many years. Very little work has been done, however, to actually measure $\overline{uv}$ and $\overline{vt}$ and experimentally substantiate the theory.

The literature contains little data other than that of Johnson in ref. 1. He obtained measurements of these and other quantities in the

1. INTRODUCTION

In the theory of statistical mechanics, the partition function Z and the free energy F are the most important quantities. The partition function Z is defined as the sum of the Boltzmann factors over all possible states of the system.

(The partition function Z is defined as)

where ϵ_i is the energy of the i -th state.

$$(1) \quad Z = \sum_i e^{-\beta \epsilon_i} = \sum_i e^{-\beta (E_i - \mu N_i)} = e^{-\beta E} e^{\beta \mu N}$$

where E is the total energy and N is the total number of particles.

$$(2) \quad F = -k_B T \ln Z = -k_B T \ln \left(\sum_i e^{-\beta \epsilon_i} \right)$$

The partition function Z is a function of the temperature T and the chemical potential μ . The free energy F is a function of the temperature T and the number of particles N . The partition function Z is a function of the temperature T and the chemical potential μ .

It is important to note that the partition function Z is a function of the temperature T and the chemical potential μ .

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The partition function Z is a function of the temperature T and the chemical potential μ . The free energy F is a function of the temperature T and the number of particles N . The partition function Z is a function of the temperature T and the chemical potential μ .

boundary layer over a flat plate with an unheated starting length. His results are given for only one vertical exploration of the boundary layer, and they agree with the expected behavior of $\overline{uv}$ and $\overline{vt}$ as functions of distance from the heated plate.

It has been proposed that a thorough investigation of the thermal boundary layer properties be undertaken at M.I.T. Therefore, it is of fundamental importance to design an instrumentation system of sufficient sensitivity and accuracy to evaluate such properties.

It was the purpose of this thesis to assemble such an instrumentation system and make preliminary measurements over a heated flat plate. Theoretically a hot-wire anemometer system will provide sufficient data for calculating $\overline{uv}$ and $\overline{vt}$. A basic system was assembled and tested. Certain components were available in the laboratory, and the remaining items were selected from commercially available instruments. Appendix D contains a description of the equipment.

boundary layer over a flat plate with no velocity gradient. The results are given for only one velocity gradient of the boundary layer, and they agree with the expected behavior of $\bar{u}$ and $\bar{v}$ as functions of distance from the heated plate.

It has been pointed out in the preceding investigation of the thermal boundary layer properties in laminar flow at $Pr = 0.7$, that the fundamental properties to design an instrumentation system of sufficient accuracy and accuracy in measuring such properties. It was the purpose of this thesis to develop such an instrumentation system and also experimentally determine over a heated flat plate. Theoretically a two-dimensional system will provide sufficient data for calculating $\bar{u}$ and $\bar{v}$. A basic system was assembled and tested. Details concerning the available in the laboratory, and the resulting data were selected from completely available instruments. Figure 1 contains a description of the apparatus.



II. PROCEDURES

The experiment was conducted in the Heat Measurement Laboratory using one of the small scale wind tunnels. The boundary layer was developed over a 40" aluminum plate resting on the bottom of the tunnel. The last twelve-inch section of the plate was heated by a coil of resistance heating wire on the underside. Measurements were made along a vertical traverse through the boundary layer ten inches from the start of the heated section.

The hot-wire anemometer equipment consisted of hot-wire probes, a Flow Corporation HWB amplifier and bridge unit and a Ballantine 320 true root-mean-square volt-meter.

Two different hot-wire anemometer probes were used. One was a single-wire; the other a V-wire probe. Appendix D describes the equipment in greater detail.

Because the V-probe soon proved unacceptable, the single-wire probe was used thereafter. Orienting the wire first at plus 45° , then at minus 45° to the free stream velocity gave the desired data. Since only the time-averaged quantities enter the calculations, data for each orientation did not need to be taken at the same time.

After thermal aging for a half-hour, the wire was carefully calibrated by recording the wire current at each resistance ratio for various wind velocities. A micromanometer was used to obtain accurate velocities.

A plot of I^2 vs $\sqrt{U}$ appears in Fig. 8 for $\gamma = 1.2$ and 1.4. These

II. EXPERIMENTAL

The experiment was conducted in the High Temperature Laboratory using one of the small cells with furnace. The furnace jacket was equipped with a 10' aluminum plate resting on the bottom of the furnace. The low resistance furnace at the side was heated by a coil of resistance heating wire on the outside. Measurements were made along a vertical traverse through the furnace, light was taken from the side of the heated section.

The hot-side measurement equipment consisted of hot-side probe, a flow meter, and a recorder and bridge unit and a resistance 300 ohm thermocouple with center.

The following hot-side measurement probes were used. One was a single-throw, the other a 2-throw probe. Appendix 9 describes the equipment in greater detail.

Between the 2-probe and 1-probe measurements, the single-throw probe was used. During the time that it was 47°, then at 48° to 49° to the flow stream velocity gave the desired data. Since only the thermocouple resistance was the calibration, data for each orientation did not need to be taken at the same time.

After several days for a full-scale, the data was carefully collected by recording the wire current at each resistance value for various wire velocities. A microammeter was used to obtain accurate voltages. A plot of I^2 vs $\sqrt{V}$ appears in fig. 2 for 1.1 and 1.2. These

curves agree with the characteristic straight lines predicted by the King equation.

From these curves it is possible to calculate I' from equation (36), where

$$I' = \frac{C_2}{4I \sqrt{U}} \quad (5)$$

As equation (72) predicts, the quantities $\overline{uv}$ and $\overline{vt}$ are related by

$$\frac{1}{2} \left(\frac{i_s}{I'} \right)^2 \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn} - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn} - M_{sn}^2} \right)_a \right] \\ = - \overline{uv} + \left\{ \frac{\sqrt{2}}{4I'} \frac{\alpha e}{1 - [1 + \alpha_e(\overline{T}_g - \overline{T}_e)]} \right\} (I_a + I_b) \overline{vt}, \quad (6)$$

where the subscripts a and b denote the quantities for the two wire orientations. It is important that the wires of whatever probe is used be correctly angled to the flow, i.e. $\pm 45^\circ$ for this case.

The procedure for measuring and calculating these quantities follows:

Ref. 12, the HWB Instruction Book, describes how to compute the square-wave current and how to set the desired resistance ratios. Briefly, the "Bridge Null" dial of the HWB is adjusted to balance the bridge for a "cold" current in the wire of about one and a half milliamps.

At one of the available resistance ratios, the current is increased in the wire until the bridge is again balanced. The values of resistance ratio and "bridge null" dial setting then enter the calculation of square-wave current as shown in Appendix E.

The following sequence of readings was obtained at the maximum tunnel velocity of 26 ft/sec with the plate first unheated, then heated:

curves agree with the characteristic straight lines predicted by the King equation.

From these curves it is possible to calculate V_1 from equation (3b),

where

$$(3) \quad V_1 = \frac{V_2}{\sqrt{1 + \frac{V_2^2}{V_1^2}}}$$

In equation (7b) predicted, the quantities v_1 and v_2 are related by

$$(4) \quad \frac{v_1}{v_2} = \frac{\left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right)}{\left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right)} \cdot \frac{\left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right)}{\left(\frac{M_{12}}{M_{22}} - \frac{M_{11}}{M_{21}} \right)} \cdot \frac{\alpha}{\sqrt{1 + \alpha^2}} \cdot \frac{1}{\sqrt{1 + \alpha^2}}$$

where the subscripts 1 and 2 denote the quantities for the two wire orientations. It is important that the wires of whatever shape is used be correctly aligned to the flow, i.e. $\frac{1}{2} \pi$ for this case.

The procedure for measuring and calculating these quantities follows:

Ref. 12, the King Instruction Book, describes how to compute the square-

wave current and how to set the desired resistance ratios. Briefly, the

"Bridge Null" dial of the flow is adjusted to balance the bridge for a

"cold" current in the wire of about one and a half milliamperes.

As one of the available resistance ratios, the current is increased in

the wire until the bridge is again balanced. The values of resistance ratio

and "Bridge Null" dial setting then enter the calculation of square-wave

current as shown in Appendix E.

The following sequence of readings was obtained at the maximum tunnel

velocity of 10 ft/sec with the plate first unstayed, then heeled:

With the wire at plus 45° , values of M_{sn} , M_{scn} and I_a were recorded at resistance ratios of 1.2 and 1.4. This was repeated at predetermined distances, y , above the plate as measured by the micro-traverse gear, starting where the probe shank touched the plate.

Values of M_{sn} , M_{scn} and I_b were subsequently recorded with the wire at minus 45° for the same values of y .

The purpose of the hot and cold plate readings becomes clear when one assumes $t = 0$ for the unheated plate. Then equation (72) is

$$\overline{uv} = -\frac{1}{2} \left(\frac{I_s}{I_t} \right)^2 \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right]. \quad (7)$$

However, with the heated plate the full equation applies.

The remaining part of the coefficient of $\overline{vt}$, $\frac{\alpha_e}{\gamma - [1 + \alpha_e(T_g - T_e)]}$, may be calculated as shown in Appendix E.

Making measurements at two resistance ratios gives two simultaneous equations at each y . Solution of these equations gives $\overline{uv}$ and $\overline{vt}$ for the hot plate, and $\overline{uv}$ for the cold plate. With the plate cold $\overline{uv}$ is over-determined, so the average of the two equations was taken.

A traverse of the boundary layer with the wire perpendicular to the mean flow gives a comparison of the wire current with that in the free stream. From the I^2 vs $\sqrt{U}$ calibration data on the wire, the average velocity as a function of y can be picked off.

Similarly, a measurement of wire currents at a position first with the plate heated, then unheated, allows calculation of the temperature rise above ambient by the method of Appendix E.

with the value of β at $\beta = 0$, which is β_{00} , and β_{00} was determined
 at equilibrium values of 1.1 and 1.4. This was repeated at predetermined
 distances, β , above the plane as measured by the micro interferometer.
 Assuming that the value of β was constant in the plane.
 Values of β_{00} and β were independently measured with the plane
 at values $\beta = 0$ for the case values of β .

The curves of the first and second plane readings become linear when one
 inserts a β for the observed value. This equation (7) is

$$(7) \quad \bar{v} = -\frac{1}{2} \left(\frac{1}{\beta} \right) \left[\left(\frac{1}{\beta} - \frac{1}{\beta_0} \right) \left(\frac{1}{\beta} - \frac{1}{\beta_0} \right) \right]$$

However, when the value of β is not constant in the plane, the
 The equation for the coefficient of β , β_0 , is
 and the calculation of β is given by equation (8).

Using measurements of the equilibrium values of β and β_0 for the
 equation of β and β_0 . Values of these equations give β and β_0 for the
 for β and β_0 for the case β_0 . The value of β is over-

estimated, as the value of β is not constant in the plane.
 A diagram of the boundary layer with the value of β is shown in the
 from the value of β and β_0 of the value of β in the first
 however, the value of β is not constant in the plane, the value
 values of β and β_0 can be given by

Similarly, a comparison of the values of β and β_0 with
 the value of β and β_0 of the value of β in the first
 where values of β and β_0 are given by

III. RESULTS

A. Figure 1 shows $\overline{uv}/U_\infty^2$ as a function of vertical distance from the plate for the heated and unheated conditions. Reasonable agreement with published data is demonstrated. The value of this quantity is greater in the heated plate condition than for the cold plate, as was noted in reference (1).

Figure 2 is a graph of the temperature $(\overline{T}_g - T_e)/(T_s - T_e)$ and velocity $(\overline{U}/U_\infty)$ profiles across the boundary layer.

Figure 3 is a plot of $\overline{uv}/U_\infty^2$ and $\overline{vt}/U_\infty (T_s - T_e)$ for the heated plate. The best data obtained is shown. The extreme scatter of the data precludes computing E_m and E_h as functions of y in order to establish an analogy between them.

B. The basic instrumentation selected includes: (a) a hot wire probe with 0.00015" tungsten wires; (b) a Flow Corporation, Hot Wire Anemometer, model HWB; and (c) a modified Ballantine Laboratory, True RMS Voltmeter, model 320. This equipment will measure accurately the properties of a momentum boundary layer. It is probable that the additional properties of a thermal boundary layer can also be detected with this instrumentation; however, this cannot be stated conclusively. Appropriate recommendations are included in section VI. The equipment is illustrated in figure 4.

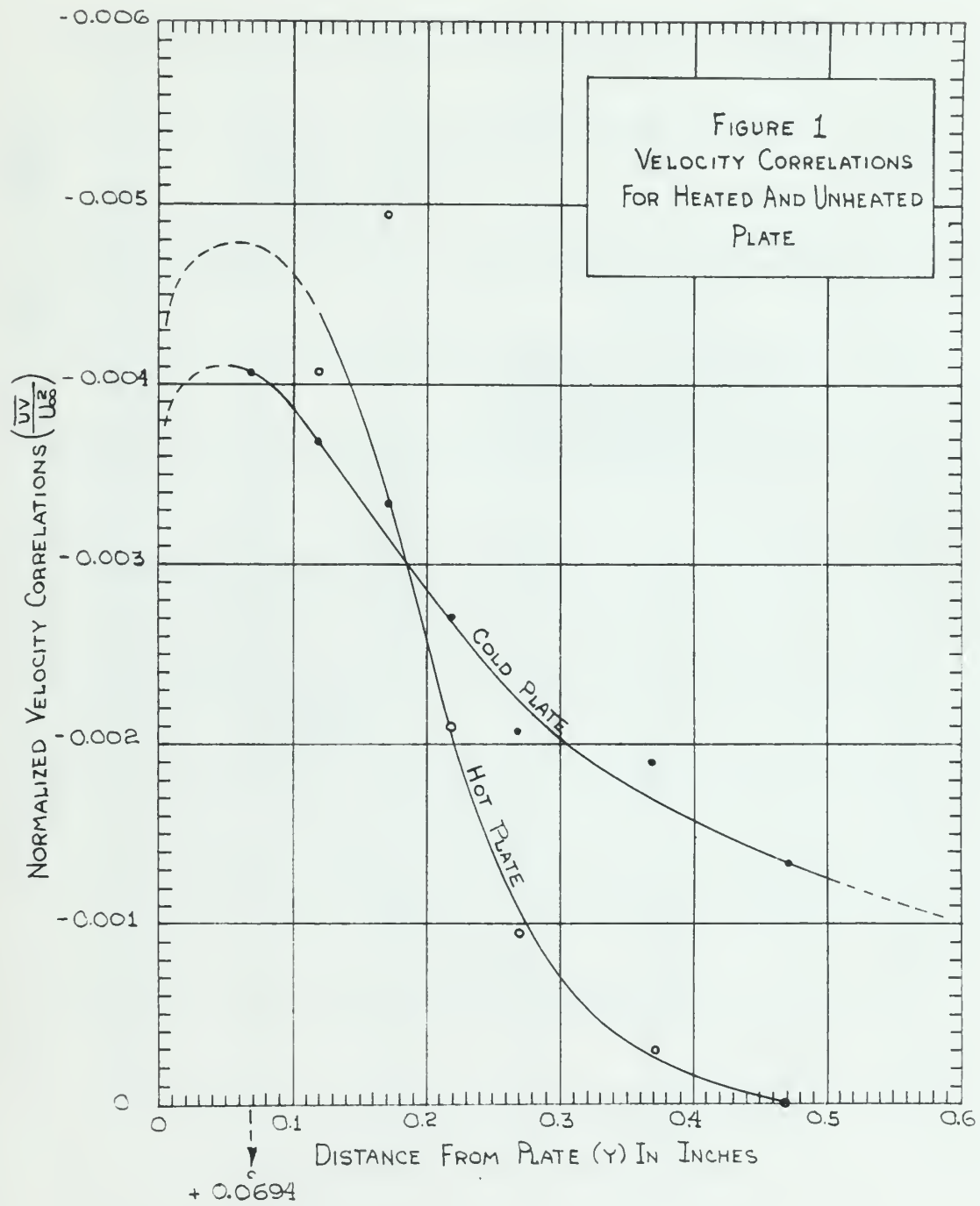
III. RESULTS

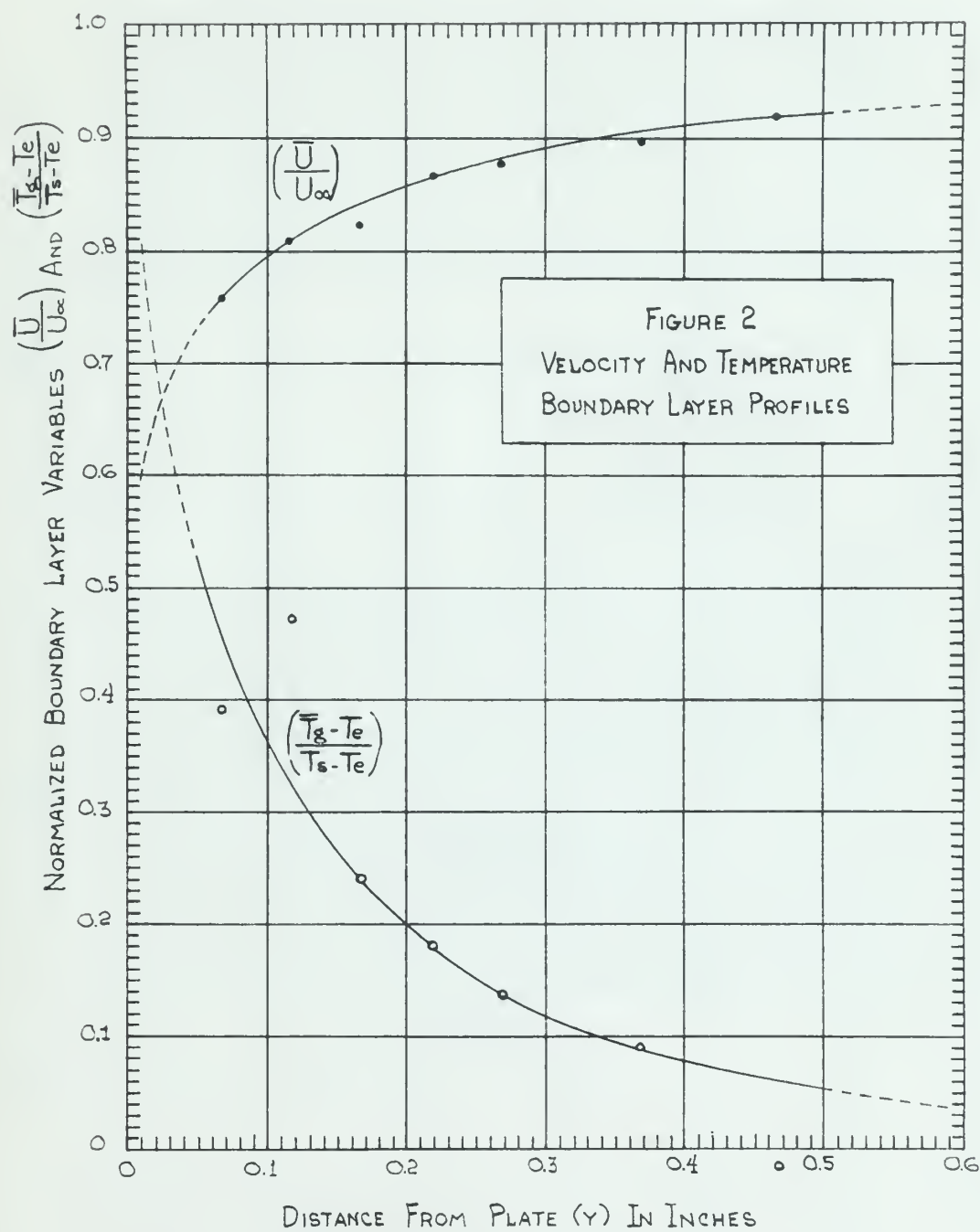
1. Figure 1 shows $\bar{v}_{\infty}$ as a function of reduced distance from the plate for the heated and insulated conditions. Reasonable agreement with published data is demonstrated. The value of this quantity is greater in the heated than insulated case for the cold plate, as was noted in reference (1).

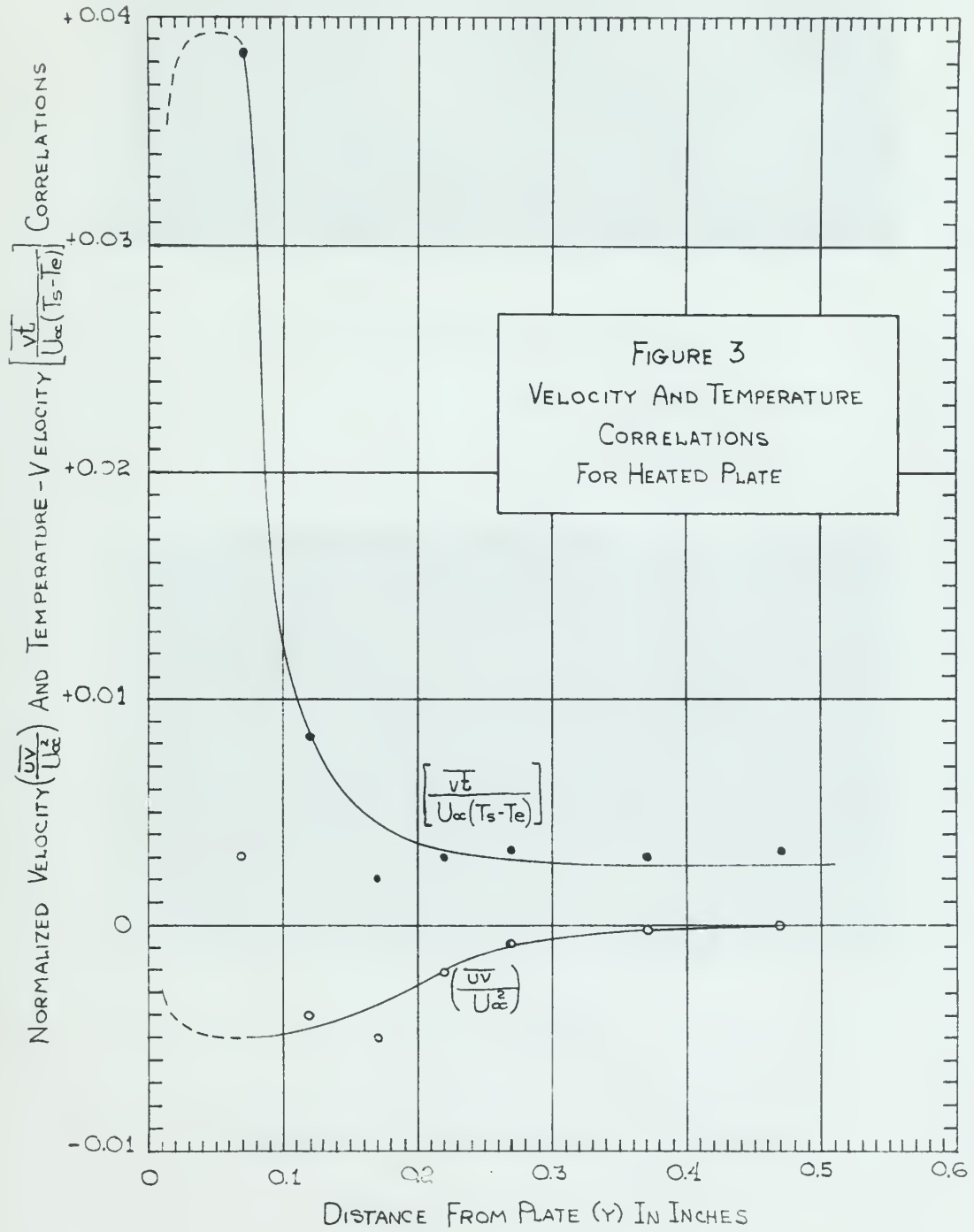
Figure 2 is a graph of the temperature $(T-T_0)/(T_1-T_0)$ and velocity (U/U_0) profiles across the boundary layer.

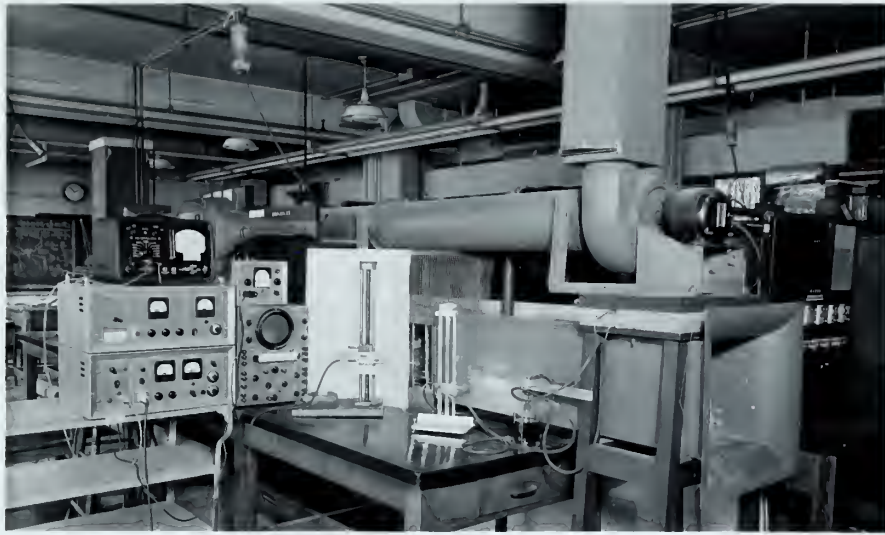
Figure 3 is a plot of $\bar{v}_{\infty}$ and $\bar{v}_{\infty}^*$ for the heated plate. The base data obtained in section II are shown. The various results of the data presented comparing $\bar{v}_{\infty}$ and $\bar{v}_{\infty}^*$ as functions of x in order to establish an analogy between them.

2. The basic heat-transfer-related results: (a) a hot wire probe with 0.00125 inch diameter; (b) a thin film sensor; and the associated model (10) and (11) (modified) calibration factors, from the following model (10). This approach will somewhat accurately the properties of a constant boundary layer. It is probable that the additional properties of a constant boundary layer can also be obtained from this instrumentation; however, this cannot be stated conclusively. Appropriate calibrations are included in section II. The system is illustrated in Figure 4.









(a). Equipment Arrangement



(b). Probes

Figure 4 - Experimental Equipment



IV. DISCUSSION OF RESULTS

A. The plot of $\overline{uv}/U_\infty^2$ in Fig. 1 has about the same order of magnitude, although somewhat higher, as that shown by Johnson in ref. 1. The slope of this curve is much greater than Johnson's however. He did obtain two or three points close to the heated plate that are much higher than those for the cold plate. Further from the plate Johnson's velocity correlation quantities are almost identical for the heated and unheated conditions.

On the other hand, the quantities shown in Fig. 1 for the heated plate remain generally higher than for the cold plate over the span measured. This difference could easily be due to experimental error, however there is a possible explanation. Thermal convection from the heated plate could be expected to disrupt the laminar sublayer causing some increase in the turbulence of the flow close to the plate. Fluid moving from the sublayer would have lower velocities and therefore tend to correlate $a(-u)$ with $a(-v)$ to add to the quantity $(-\overline{uv})$ for the cold plate.

B. The plots of $\overline{U}/U_\infty$ and $(T_g - T_e)/(T_s - T_e)$ as a function of distance from the plate show the typical characteristics of these functions. The velocity profile indicates that measurements should be taken closer to the plate to further delineate the shape of the curve. To do this a much shorter hot-wire would be needed.

IV. MINIMUM OF ERROR

A. The plot of $\overline{u^2}/V_0^2$ in Fig. 1 was shown the same order of magnitude, although somewhat higher, as that shown by Johnson in ref. 1. The slope of this curve is much greater than Johnson's however. He did obtain two or three points closer to the heated plate than any other than those for the cold plate. Further from the plate Johnson's velocity measurements quantities are almost identical for the heated and unheated conditions. On the other hand, the quantities shown in Fig. 1 for the heated plate remain generally higher than for the cold plate over the span measured. This difference could easily be due to experimental error, however there is a possible explanation. Thermal convection from the heated plate could be expected to disturb the laminar velocity causing some increase in the turbulence at the plate and the plate. This moving from the surface would have lower velocities and therefore tend to correlate $u(-y)$ with $u(y)$ so that the quantity $(-uv)$ for the cold plate.

B. The plots of $\overline{u^2}/V_0^2$ and $(T-T_0)/(T-T_0)$ as a function of distance from the plate show the typical characteristics of these functions. The velocity profile indicates that measurements should be taken closer to the plate to further determine the shape of the curve. To do this a much steeper slope would be needed.

Extrapolating the curve of $T_g - T_e$ in Fig. 10 to the plate gave an estimated plate surface temperature, T_s , of 8°C above ambient. Such a low temperature difference made temperature measurements difficult.

Another more accurate method of measuring temperature would be to record the bridge null at "cold current" (about one and a half milliamps) while traversing the boundary layer. For this a bridge galvanometer amplifier is required to enable more accurate bridge null settings than is possible with this model HNE. The later HNE models do incorporate such an amplifier.

C. As stated in the Results, the scatter in the computed points $\overline{v_t}/U_\infty (T_s - T_e)$ in Fig. 3 did not warrant calculation of E_m and E_h by equations (3) and (4). This is one of the most important ultimate goals of these measurements that was not realized in this thesis. No reliable curve of $\overline{v_t}$ can be inferred, and several attempts to deduce better values of this quantity produced no better results than those shown. A higher plate temperature would likely give larger, more-easily-measured quantities.

The data does not show the predicted behavior of $\overline{uv}$ and $\overline{v_t}$ going to zero at the plate. At the closest approach, the center of the wire is still 0.068 inches from the plate. Any decrease in these quantities must therefore be inside this distance.

D. The equations (69) and (70) in Appendix B for two wires apply also for wires mounted on an X-probe. Johnson used an X-probe with both wires hot for detecting velocity fluctuations and a single cold wire close by for temperature variations. The hot-wire theory predicts that it is unnecessary to use more than one hot-wire. Since equation (71) has five unknowns, it is necessary only to make measurements with one wire at five resistance ratios and solve the resulting equations simultaneously.

If two wires on an X-probe are controlled independently by two HWB's with outputs measured on separate meters, equations (69) and (70) may be solved together at five resistance ratios to yield the boundary layer properties. The two important correlated quantities are dominant in equation (72).

The practice in this experiment of orienting the probe at $\pm 45^\circ$ simulated an X-probe and eliminated the problem of matching the two wires to close tolerances. Experience with the V-probe supports the comment made by Johnson (p. 54 of ref. 1) that it is unlikely I' , I , α_e , the time constant, length, diameter and other characteristics of two wires can ever be matched simultaneously.

A few comments on the V-probe may warn others of the pitfalls of its peculiarities.

It was first discovered that the resistance of the common current lead and center needle is unavoidably in one arm of the bridge circuit. Since the voltage across the center lead is proportional to the sum of the two wire currents, the bridge null for one wire is affected by current in the other.

The second problem is serious, particularly in a thin boundary layer. The two wires are sufficiently separated to be in different stream conditions and do not correlate quantities at a point.

The third problem has been discussed - that of matching any two wires.

Finally, several times it was noted that wires on the V-probe were not taut and straight several hours after rewiring. This means that wires are not at the specified angle to the stream.

E. One problem that detracts from the accuracy of the results is ambient temperature variations while the measurements are being made in a traverse

It was found that the temperature was controlled independently by the heater
with separate heaters for separate wires, equations (5) and (7) say
together at five resistance ratios in the boundary layer
properties. The two important material properties are defined in
equation (7).

The analysis in this paper is of a system of wires at $\frac{1}{2}$ cm
laced at 1-cm and estimated the position of maximum temperature to
give a maximum. The temperature at the 1-cm wire is the lowest
by Jensen (p. 35 of ref. 1) that it is minimum, i.e., α , the time
constant, length, diameter, and other characteristics of the wire can vary
as required.

A few comments on the 1-cm wire can be made on the basis of its
properties.

It was first discovered that the resistance of the common circuit
load and center wire is considerably in excess of the bridge circuit.
Since the voltage across the center load is proportional to the sum of
the two wire resistances, the bridge will not be affected by current
in the other.

The second problem is serious, particularly in a thin boundary layer.
The two wires are sufficiently separated so as to be in different stream condi-
tions and do not represent quantities at a point.

The third problem has been discussed - that of reading any two wires.
Finally, several times it was noted that wires on the 1-cm wire
and that and several other wires were also reading. This means that wires
are not at the specified height in the stream.

One problem that arises from the accuracy of the results is related
temperature variations while the measurements are being made in a stream.

through the boundary layer. Using the minimum number of two resistance ratios at each y , it required four to six hours to take data at seven points. Since the ambient temperature can change by several degrees during this time, it is desirable to have a constant-temperature air input to the tunnel.

It was noticed that there was considerable velocity fluctuation in the free stream on the axis of the tunnel. This masked the "edge" of the momentum boundary layer.

To reduce the above defects of this tunnel it is desirable that the entrance be fitted with a plenum where the air can be heated a few degrees above the ambient at the same time the velocity fluctuations are being smoothed.

Along with tunnel entrance modifications other test facility improvements should be incorporated.

Although a complete survey of the boundary layer should include measurements at various distances along the plate, it would be well for the uninitiated to practice in boundary layers at least two inches thick. This means having an unheated starting length of at least 40 inches long and a heated section 30 inches long. The tunnel should be three to four feet wide to approximate an infinitely wide plate.

The heated plate should be well-insulated on the bottom and equipped with heating coils and thermocouples for obtaining a constant (q/Λ) or constant temperature plate as desired.

F. Of all the problems encountered in this experiment, that of measuring accurate (i.e. less than 5%) true root-mean-square voltages caused the most consternation. The best available information indicated that the Ballantine Model 320 and Flow Corporation TBM were the only two meters commercially available reputed to measure RMS voltages on a random signal.

through the boundary layer. Using the distance between the two electrodes
of about 1 cm. it was found that the rate of flow is about 10 cm. per hour.
From the subject's position the change in pressure is about 10 mm. Hg.
It is therefore to be seen that a considerable amount of work is done.
It was noticed that there was considerable resistance to the flow
from the air in the lungs. This was the "leak" of the
respiratory system.
To reduce the above defects of this device it is desirable that the
distance be fixed at a point where the air can be forced a few degrees
from the subject at the point where the velocity of flow is very
great.
Along with these defects certain advantages of this device are
also to be mentioned.
Although a complete record of the boundary layer should include
measurements at various distances from the lungs it would be well to
the subject to breathe in through the lungs at least two inches from
the mouth being an average distance of about 10 inches long
and a distance of about 10 inches from the lungs. The subject should be able to
breathe in through the lungs at least two inches from the mouth.
The device should be well-lubricated on the inside and equipped
with heating coils and thermometers for obtaining a constant (40°) or
constant temperature of the air.
Of all the problems mentioned in this report, that of measuring
the rate of flow is the most difficult. The best available method is the
method of the flow meter. The rate of flow is measured by the volume of
air which is forced out of the lungs in a given time.
The rate of flow is measured by the volume of air which is forced out of the lungs in a given time.

When the short time constant of the Model 320 was lengthened, it became the most reliable instrument. However, both meters exhibited the annoying characteristic of indicating different values of the same voltage on adjacent scales. The TBM became unusable for this reason, and the Model 320 gave ambiguous readings only at times. The limited "crest factor" of the Model 320 could permit peak clipping of the signal on the lower scale and thereby alter the waveform. However, such should not be the case for the TBM with its much higher "peak factor."

This problem was not resolved and is the most likely cause of the wide scatter in the calculated results. Knowing that this problem existed provided the hint to smooth the data as a function of y before re-calculating $\overline{uv}$ and $\overline{vt}$. Even so the improvement was not marked.

It is interesting to note that the literature reviewed for this thesis makes no mention of true RMS measurements or time-averaged readings in similar work.

The Model 320 could be improved by making a well-designed permanent modification to obtain a long time constant, mean square averaging circuit. If the meter time constant cannot be suitably controlled, then a five to ten minute time recording of the output could be graphically integrated to establish the best value of each voltage.

A more elaborate scheme of calibrating the meter is needed. The most appropriate method would be to use a variable-frequency generator with a calibrated output adjustable from one millivolt to one volt. It could be used to resolve the problem of peak clipping of the signals.

G. One test of accurate data is its repeatability. Time permitted little experimenting along this line, but it was often noted that the bridge could be rebalanced and currents reset to a maximum variation of one part in 150 or 0.7%.

When the short time constant of the input 320 was investigated, it became the most reliable instrument. However, high speed exhibited the varying characteristics of indicating different values of the same voltage on adjacent scales. The two scales available for this reason, and the input 320 gave ambiguous readings only at times. The first "approximate" of the input 320 could result from clipping of the signal on the lower scale and thereby alter the waveform. However, such should not be the case for the first with the much higher "peak factor".

That output was not twisted and is the more likely cause of the many errors in the calculated results. Knowing that this problem existed provided the data as shown in the table as a function of γ before re-calculating the data. Even so the improvement was not sufficient.

It is interesting to note that the literature reviewed for this thesis makes no mention of any of the measurements or time-constant problems in similar work.

The input 320 could be improved by making a self-designed instrument modification to provide a long time constant, with square wave input circuit. If the input time constant cannot be suitably controlled, then a time to for about the frequency of the output could be practically impossible to establish the first value of each voltage.

A more elaborate scheme of calibrating the input is needed. The most appropriate method would be to use a variable-frequency generator with a calibrated output adjustable from one millivolt to one volt. It could be used to transfer the problem of peak clipping of the signal.

6. The best of economic data is the responsibility. Two parameters have been determined along this line, but it was often noted that the bridge could be replaced and current used in a similar manner as one part in 100

In the presence of signal fluctuations the galvanometer fluctuated also. This was particularly noticeable at high resistance ratios. In setting average currents it became necessary to average the needle fluctuations by eye. Occasionally this led to variations of one part in 75 or 1.3%. Provision of a selectable time constant on the galvanometer would assist in setting more accurate currents and resistance ratios.

At the time they were taken, meter readings could be averaged within 2% in spite of the slow fluctuations. However, it was rarely possible to repeat the data at a point because ambient conditions had changed. On the few occasions when it was possible to check results, meter readings were well within 5% barring anomalies already discussed.

In the present of rapid fluctuations the Government is situated also.

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 way. The fluctuations are not so regular as was said in 1911. The
 relation of a regularity like movement on the Government would result in
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At the time that was taken, when the fluctuations could be stopped within
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 the fluctuations that it was possible to stop within, when the fluctuations were
 well within the fluctuations which were not possible.

V. CONCLUSIONS

Hot wire anemometer theory predicts that both of the desired correlation quantities, $\overline{uv}$ and $\overline{vt}$, can be obtained with a single, matched, X-wire probe. Due to the extremely small volume affected by the fluctuation quantities at any instant, velocity-temperature sensing elements in a single probe is preferable to separate probe sensing elements.

The instrumentation system, modified as recommended in section VI, should yield data of sufficient accuracy for calculation of all the thermal boundary layer properties. The results presented do not affirm this statement without question, but they do indicate that further careful investigation is warranted. Refinement of instrument sensitivity and experimental technique are the paramount requisites.

7. DISCUSSION

For very low-frequency signals the value of the relative error, $\overline{\epsilon}$, can be calculated with a single, rather, X-ray probe, but in the frequency range covered by the frequency spectrum in our studies, relative error is calculated in a single probe is calculated in separate probe sections.

The representation of the calculated error is presented in section VI, should take into account the calculation of all the chemical boundary layer properties. The results presented in our studies show that the error is small, but they do indicate that further careful investigation is warranted. Estimates of instrument sensitivity and experimental techniques are the present requirements.

VI. RECOMMENDATIONS

The following modifications of instrumentation and facilities are required for accurate measurements of the thermal boundary layer properties:

- (1) Permanent modification of the meter circuit of a Ballantine 320 to obtain a long time constant averaging period;
- (2) Accurate calibration of all the range scales on the voltmeter by use of an accepted standard frequency meter which has a calibrated voltage output;
- (3) Construction of an X-wire probe with the wires oriented accurately at 90° to each other and their resistance matched within 2%;
- (4) Construction of a longer heated section for the plate to increase the boundary layer thickness;
- (5) Construction of a larger scale wind tunnel with an inlet heated plenum chamber to insure a constant temperature, constant velocity air stream in the tunnel;
- (6) Procurement of an additional HWB, modified to include a sum and difference unit in lieu of the amplifier section.

In addition, the data processing techniques should be refined as follows:

- (7) An averaging technique must be used over and above that which is provided in (1) above - e.g., averaging of sequentially timed observations, or graphical integration of the recorded output of the voltmeter;
- (8) All five resistance ratios permitted by the wire size must be used

(6) All test specimens shall be prepared by the same method as used

the following:

specimens, or graphical representation of the recorded weight of

provided in (1) above - e.g., averaging of specimens) shall be

(7) In averaging specimens used in test over and above that which is

as follows:

In addition, the data representing specimens shall be reduced

differences shall be less than the specified amount.

(8) Treatment of an individual test, specified in Section 4.0.0.0.0

are shown in the summary.

specimens shall be treated as separate specimens, separate subject

(9) Construction of a larger scale than shown with no other basis

and boundary layer thickness;

(a) Construction of a larger scale section for the same or different

at 90° to each other and their respective sections shall be

(b) Construction of an L-shaped section with the other section similarly

within contact;

use of an accepted standard frequency which has a calibrated

(c) Accurate calibration of all the range scales on the specimens;

to which a long time constant frequency meter;

(1) Treatment and calibration of the data shall be as follows:

properties:

regarding the accuracy of the initial boundary layer

the following conditions of measurement and facilities are

VI. CONCLUSIONS

to calculate all five terms of equation # 7/ until their order of magnitude is firmly established. Programming of these equations for computer use would considerably speed data analysis.

for children all the time of year. It is not only a place of
recreation for the children but also a place of instruction.
The children are taught to read and write and to do arithmetic.

APPENDIX A

NOMENCLATURE

A	Cross-sectional area.
c	Heat capacity of wire.
C_p	Heat capacity of fluid at constant pressure.
C_1, C_2	Constants.
D	Wire diameter.
D^*	Wire diameter, units of 0.001 inch.
e	Voltage (general).
e_c	Output voltage from the amplifier due to square-wave current alone imposed on the wire.
e_{cn}	Output voltage from the amplifier due to square-wave current plus system noise.
e_n	Output voltage from the amplifier due to system noise.
e_s	Output voltage from the amplifier due to velocity fluctuations on the hot wire.
e_{sc}	Output voltage from the amplifier due to velocity fluctuations plus square-wave current.
e_{scn}	Same as e_{sc} plus system noise.
e_{sn}	Same as e_s plus system noise.
e_w	Hot-wire voltage before amplification.
F	Function expressing cooling effect on hot wire by velocity and pressure of stream.
F'	Defined as $\left. \frac{\partial F}{\partial V} \right _P$, the derivative of F with respect to velocity at constant P.
$\bar{F}$	Time-average value of F.

- f Instantaneous deviation value of F from $\bar{F}$.
- H Rate of heat loss from the hot wire per unit length.
- h Fluid enthalpy per unit mass.
- I Hot-wire DC current.
- $\bar{I}$ Time-average of I .
- I' Defined as $\left. \frac{\partial I}{\partial v_p} \right|_p$, the derivative of hot-wire current with respect to velocity at constant pressure.
- i Instantaneous deviation value of I from $\bar{I}$.
- i_s Instantaneous deviation value of hot-wire current due to imposed square-wave.
- I_a, I_b Hot-wire DC current of wires a and b.
- J_1 Amplifier multiplier coefficient of e_w .
- J_2 Amplifier multiplier coefficient of time derivative of e_w .
- K_1 Meter constant.
- k Coefficient of thermal conductivity.
- L Length of hot-wire.
- M Meter reading of true RMS voltage.
- $M_c, M_{cn}, M_n, M_s, M_{sc}, M_{scn}, M_{sn}$ Meter readings corresponding to voltages, e , with same subscripts.
- N_{Pr} Prandtl number, equals $c_p \mu / k$.
- N_{Re} Reynolds number, equals $\mu \bar{U} D / \rho$.
- P Pressure of fluid in the boundary layer.
- $\bar{P}$ Time-average of P .
- p Instantaneous deviation value of P from $\bar{P}$.
- P^* Pressure of fluid, units of atmospheres.
- q Rate of heat transfer per unit time.
- R Hot-wire resistance.
- $\bar{R}$ Time-average of R .

r	Instantaneous deviation value of hot-wire resistance from $\bar{R}$.
R_e	Resistance of wire in equilibrium with ambient fluid temperature, T_e .
$\bar{R}_e$	Time-average value of R_e .
r_e	Instantaneous deviation value of R_e from $\bar{R}_e$.
R_g	Resistance of wire in equilibrium with fluid temperature, T_g .
$\bar{R}_g$	Time-average value of R_g .
r_g	Instantaneous deviation value of R_g from $\bar{R}_g$.
R_o	Resistance of wire at reference temperature, T_o .
T	Temperature.
$\bar{T}$	Time-average of T .
t	Instantaneous deviation value of T from $\bar{T}$.
T_e	Ambient temperature of the stream entering the tunnel.
T_g	Temperature of fluid at a point in the boundary layer.
$\bar{T}_g$	Time-average value of T_g .
T_o	An arbitrary reference temperature for computing resistance of wire material, handbook value.
T_s	Surface temperature of the heated plate.
T_w	Hot-wire temperature.
T	Temperature of the fluid in the free stream.
U	Velocity of the fluid in the x-direction.
$\bar{U}$	Time-average value of U .
u	Instantaneous deviation value of U from $\bar{U}$.
U	Velocity of the fluid in the free stream.
V	Velocity of the fluid in the y-direction.
$\bar{V}$	Time-average of V .
v	Instantaneous deviation value of V from $\bar{V}$.

1. Instantaneous deviation value of x from $\bar{x}$.
 2. Deviation of x in relation to the mean $\bar{x}$.
 3. Time-average value of x .
 4. Instantaneous deviation value of y from $\bar{y}$.
 5. Deviation of y in relation to the mean $\bar{y}$.
 6. Time-average value of y .
 7. Instantaneous deviation value of z from $\bar{z}$.
 8. Deviation of z in relation to the mean $\bar{z}$.
 9. Time-average value of z .
 10. Instantaneous deviation value of t from $\bar{t}$.
 11. Deviation of t in relation to the mean $\bar{t}$.
 12. Time-average value of t .
 13. Instantaneous deviation value of u from $\bar{u}$.
 14. Deviation of u in relation to the mean $\bar{u}$.
 15. Time-average value of u .
 16. Instantaneous deviation value of v from $\bar{v}$.
 17. Deviation of v in relation to the mean $\bar{v}$.
 18. Time-average value of v .
 19. Instantaneous deviation value of w from $\bar{w}$.
 20. Deviation of w in relation to the mean $\bar{w}$.
 21. Time-average value of w .

V_A, V_B	Velocity of fluid perpendicular to wires A and B.
V_θ	Velocity of fluid at angle θ from perpendicular to wire.
V_p	General velocity perpendicular to wire.
v_p	General deviation value of V_p from its time-average.
V^*	Velocity of fluid in units of 100 ft/sec.
W	Velocity of fluid in x-direction.
$\bar{W}$	Time-average value of W .
W_1	Rate at which work is done on a particle of fluid due to external fields.
W_f	Rate at which work is done against viscous stresses.
X, Y, Z	Body forces in the x-, y-, and z-directions.
x, y, z	Distances in the direction of mean flow, perpendicular to the plate and transverse to the plate.
α_e	Temperature coefficient of resistivity at T_e .
α_g	Temperature coefficient of resistivity at T_g .
α_0	Temperature coefficient of resistivity at T_0 .
α	Thermal diffusivity, equals $k/\rho c_p$.
γ	Resistance ratio, resistance of hot wire to resistance of cold wire.
E_h	Coefficient of eddy heat (or energy) transfer.
E_m	Coefficient of eddy momentum transfer.
θ	Angle between perpendicular to wire and mean velocity vector.
μ	Viscosity.
ν	Kinematic viscosity.
ρ	Density.
τ	Time.

Order	Item	Quantity	Unit Price	Total
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Appendix B

BOUNDARY LAYER EQUATIONS

The general equations for viscous, constant-property fluid flow are quoted below. The Navier-Stokes equations of momentum and continuity in rectangular coordinates are:

Momentum:

$$\begin{aligned}\rho \frac{DU}{Dt} &= -\frac{\partial P}{\partial x} + \rho X + \mu \left[\nabla^2 U + \frac{1}{3} \frac{\partial}{\partial x} (\nabla \cdot \vec{V}) \right] \\ \rho \frac{DV}{Dt} &= -\frac{\partial P}{\partial y} + \rho Y + \mu \left[\nabla^2 V + \frac{1}{3} \frac{\partial}{\partial y} (\nabla \cdot \vec{V}) \right] \\ \rho \frac{DW}{Dt} &= -\frac{\partial P}{\partial z} + \rho Z + \mu \left[\nabla^2 W + \frac{1}{3} \frac{\partial}{\partial z} (\nabla \cdot \vec{V}) \right]\end{aligned}\quad (8)$$

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho U) + \frac{\partial}{\partial y} (\rho V) + \frac{\partial}{\partial z} (\rho W) = 0 \quad (9)$$

The energy equation is:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + W_i + W_f = \rho \frac{Dh}{Dt} - \frac{DP}{Dt} \quad (10)$$

where W_i is the rate at which work is done on a particle of fluid by an external field and W_f is the rate at which work is done against viscous stresses. Also, D/Dt represents the substantial derivative defined as

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

The Navier-Stokes equations are simplified by assuming two-dimensional, steady, incompressible flow in the absence of body forces for an ideal fluid of constant viscosity and thermal conductivity. Prandtl's order-of-magnitude treatment further reduces them until the result is:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 U}{\partial y^2} \quad (11)$$

APPENDIX A

CONTINUITY EQUATION

The general equation for mass, momentum, and energy balance in a control volume is given by the following equations of continuity and momentum in vectorial notation:

Continuity:

$$\rho \frac{D\rho}{Dt} = - \left(\frac{\partial \rho}{\partial x} u + \frac{\partial \rho}{\partial y} v + \frac{\partial \rho}{\partial z} w \right) + \rho (\nabla \cdot \vec{V})$$

$$\rho \frac{Dv}{Dt} = - \left(\frac{\partial \rho}{\partial x} v + \frac{\partial \rho}{\partial y} w + \frac{\partial \rho}{\partial z} u \right) + \rho (\nabla \cdot \vec{V})$$

$$\rho \frac{Dw}{Dt} = - \left(\frac{\partial \rho}{\partial x} w + \frac{\partial \rho}{\partial y} u + \frac{\partial \rho}{\partial z} v \right) + \rho (\nabla \cdot \vec{V})$$

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

The energy equation is:

$$\rho \left(\frac{D\epsilon}{Dt} + \frac{1}{\rho} \left(\frac{\partial \epsilon}{\partial x} u + \frac{\partial \epsilon}{\partial y} v + \frac{\partial \epsilon}{\partial z} w \right) \right) = \frac{D\rho}{Dt} - \frac{D\rho}{Dt}$$

where ϵ is the total energy per unit mass, ρ is the mass density, and $\vec{V}$ is the velocity vector. The terms in parentheses represent the divergence of the energy flux vector, and the term $\frac{D\rho}{Dt}$ represents the material derivative of the density.

$$\frac{D\epsilon}{Dt} = \frac{\partial \epsilon}{\partial t} + \frac{\partial \epsilon}{\partial x} u + \frac{\partial \epsilon}{\partial y} v + \frac{\partial \epsilon}{\partial z} w$$

The Navier-Stokes equations are simplified by assuming two-dimensional, steady, incompressible flow in the absence of body forces. The resulting equations of continuity and momentum are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (12)$$

for application to the boundary layer of a flat plate.

Similarly, the energy equation can be written in a simpler form as

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial U}{\partial x} \right)^2 \quad (13)$$

The last term, $\frac{\mu}{\rho c_p} \left(\frac{\partial U}{\partial x} \right)^2$, is usually considered negligible.

Thus the three equations of importance are:

Momentum:

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 U}{\partial y^2} \quad (14)$$

Continuity:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \quad (15)$$

Energy:

$$U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (16)$$

with boundary conditions

$$(a) \text{ at } y=0 : T=T_s \text{ or } \frac{\partial T}{\partial y} = - \frac{q}{kA} \text{ and } U=0 \quad (17)$$

$$(b) \text{ at } y=\infty : T=T_\infty \text{ and } U=U_\infty$$

Multiplying the continuity equation by U , adding the result to the momentum equation; and multiplying the continuity equation by T , adding the result to the energy equation, then simplifying both gives

$$\frac{\partial}{\partial x} (P + \rho U^2) = \frac{\partial}{\partial y} \left(\mu \frac{\partial U}{\partial y} - \rho UV \right) \quad (18)$$

$$\frac{\partial}{\partial x} (UT) = \frac{\partial}{\partial y} \left(\alpha \frac{\partial T}{\partial y} - VT \right) \quad (19)$$

and

(11)

$$0 = \frac{V_6}{\gamma_6} + \frac{U_6}{x_6}$$

for application to the boundary layer of a thin plate.

Similarly, the energy equation can be written in a similar form as

(12)

$$\left(\frac{U_6}{x_6}\right) \frac{1}{\gamma_6} + \frac{T_6}{\gamma_6} \frac{1}{\gamma_6} = \frac{T_6}{\gamma_6} V + \frac{T_6}{\gamma_6} U$$

The last term, $\left(\frac{U_6}{x_6}\right) \frac{1}{\gamma_6}$, is usually neglected negligible.

Thus the above equation is important and

Simplification

(13)

$$\frac{U_6}{x_6} + \frac{T_6}{\gamma_6} \frac{1}{\gamma_6} = \frac{U_6}{x_6} + \frac{U_6}{x_6}$$

Continuity:

(14)

$$0 = \frac{V_6}{\gamma_6} + \frac{U_6}{x_6}$$

Energy:

(15)

$$\frac{T_6}{\gamma_6} \frac{1}{\gamma_6} = \frac{T_6}{\gamma_6} V + \frac{T_6}{\gamma_6} U$$

with boundary conditions

(16)

$$0 = U \text{ and } \frac{T}{K} = \frac{T_6}{\gamma_6} \text{ or } T = T_6 : 0 = \gamma \text{ to } (a)$$

$$\infty U = U \text{ and } T = T_6 : \infty = \gamma \text{ to } (b)$$

Multiplying the continuity equation by γ , adding the result to themomentum equation and multiplying the continuity equation by T , adding

the result to the energy equation, then simplifying now gives

(17)

$$\left(\frac{U_6}{x_6} - \frac{U_6}{\gamma_6} \frac{1}{\gamma_6}\right) \frac{1}{\gamma_6} = \left(\frac{U_6}{x_6} + \frac{U_6}{x_6}\right) \frac{1}{\gamma_6}$$

(18)

$$\left(T_6 - \frac{T_6}{\gamma_6} \frac{1}{\gamma_6}\right) \frac{1}{\gamma_6} = \left(T_6\right) \frac{1}{\gamma_6}$$

In turbulent flow Reynolds assumed that all quantities are made up of an average value plus a superposed fluctuation component such that

$$\begin{aligned} U &= \bar{U} + u \\ V &= \bar{V} + v \\ T &= \bar{T} + t \\ P &= \bar{P} + p \end{aligned} \quad (20)$$

Substituting these in equation (18) and (19), multiplying out and taking time averages:

$$\frac{\partial}{\partial x} (\bar{P} + \rho \bar{U}^2 + \rho \bar{u}^2) = \frac{\partial}{\partial y} \left(\mu \frac{\partial \bar{U}}{\partial y} - \rho \bar{U} \bar{V} - \rho \bar{uv} \right) \quad (1)$$

$$\frac{\partial}{\partial x} (\bar{U} \bar{T} + \bar{ut}) = \frac{\partial}{\partial y} \left(\alpha \frac{\partial \bar{T}}{\partial y} - \bar{V} \bar{T} - \bar{vt} \right) \quad (2)$$

In general $\bar{uv}$ and $\bar{vt}$ may be expected to have values other than zero as argued below: The $\bar{U}$ velocity profile in a boundary layer over a flat plate is as shown in the sketch, Fig. 5. Choosing a point at x, y : If a 'packet' of air goes by the point in a $+V$ direction, air from closer to the plate will be forced into the 'hole' left by the packet. On the average, the entering air will have a U -velocity less than that which is average for the point x, y . This corresponds to a $(-u)$ as observed at that point.

Conversely, movement of a packet in the $(-v)$ direction gives rise to a $(+u)$ velocity.

Consequently, where there is a gradient of $\bar{U}$, a $(+v)$ produces a $(-u)$ and a $(-v)$ produces a $(+u)$, therefore the product is negative on the average. u and v are said to be 'cross-correlated' such that

$$\bar{uv} = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (uv) d\tau \quad (23)$$

In Cartesian flow Reynolds number $Re = \rho U L / \mu$ is defined as

of an average value $\bar{u}$ and a characteristic length L is given by

$$Re = \bar{u} L / \nu$$

$$Re = \bar{u} L / \nu$$

(10)

$$Re = \bar{u} L / \nu$$

$$Re = \bar{u} L / \nu$$

Substituting $\bar{u}$ from equation (10) and (11), multiplying out and taking

the average

$$(1) \quad \left(\bar{u} \bar{q} - \bar{u} \bar{q} - \frac{\bar{u} \bar{q}}{\gamma^6} \right) \frac{\gamma^6}{\gamma^6} = \left(\bar{u} \bar{q} + \bar{u} \bar{q} + \bar{q} \right) \frac{\gamma^6}{\gamma^6}$$

$$(2) \quad \left(\bar{u} \bar{q} - \bar{u} \bar{q} - \frac{\bar{u} \bar{q}}{\gamma^6} \right) \frac{\gamma^6}{\gamma^6} = \left(\bar{u} \bar{q} + \bar{u} \bar{q} \right) \frac{\gamma^6}{\gamma^6}$$

In general $\bar{u}$ and $\bar{q}$ may be expected to have values other than zero

as shown below. The $\bar{u}$ velocity profile in a boundary layer over a flat

plate is as shown in the sketch, Fig. 2. The $\bar{q}$ velocity profile in a

'pocket' of air near the point is $u + v$ direction, air from closer to

the plate will be forced into the 'pocket' by the pocket. As the flow

goes, the velocity will have a u -velocity less than that which is shown

also for the case u, v . This corresponds to $u - v$ as observed in this

case.

Consequently, instead of a pocket in the $(-v)$ direction there will

be a $(+v)$ velocity.

Consequently, where there is a gradient of u , a $(+v)$ velocity is

$(-u)$ and a $(-v)$ velocity is $(+u)$. However the product is negative in

the average. u and v are said to be 'cross-correlated' when this

$$(11) \quad \overline{uv} = \lim_{\gamma \rightarrow \infty} \frac{1}{\gamma} \int_{-\gamma}^{+\gamma} (uv) d\gamma$$

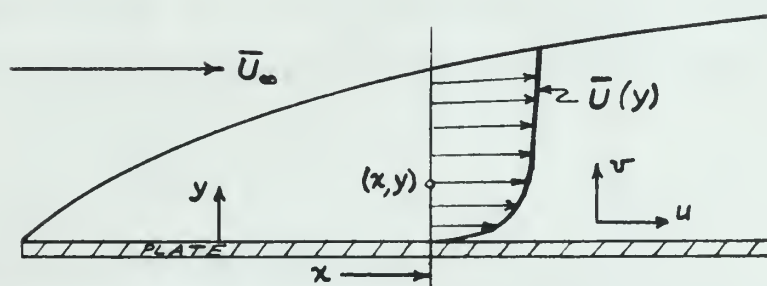


Fig. 5. Velocity Profile

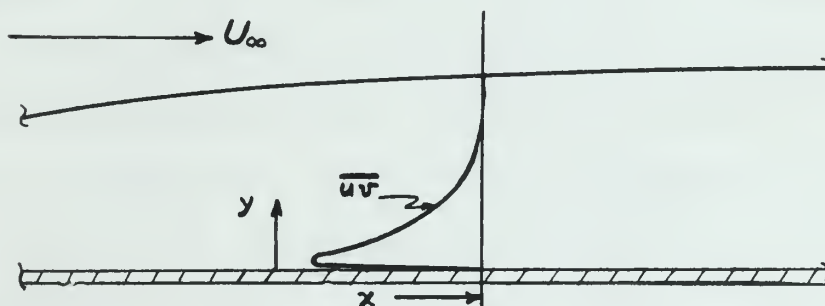


Fig. 6. Velocity Correlation Profile

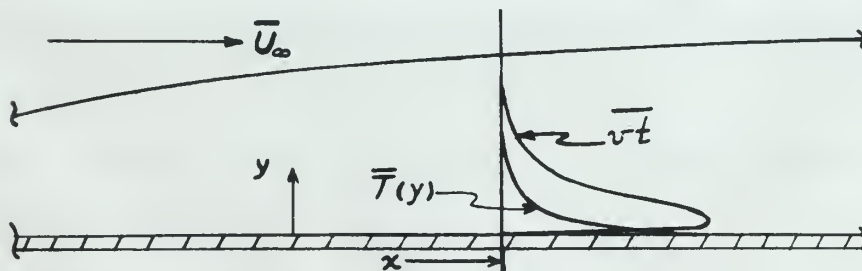


Fig. 7. Temperature and Velocity-Temperature Correlation Profiles

Close to the plate the fluid becomes laminar in the sub-layer, so u and v tend to zero. At the edge of the boundary layer where the gradient of U disappears, u and v tend to cross-correlate in a random fashion so the average is zero.

In summary, the expected behavior of $\overline{uv}$ as a function of y should appear as shown in Fig. 6. Parallel reasoning leads one to expect a positive correlation of $\overline{vt}$ where there is a gradient of $\overline{T}$ such that

$$\overline{vt} = \lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (vt) d\tau \quad (24)$$

These results are sketched in Fig. 7.

There is also a gradient of temperature in the x -direction producing a $\overline{ut}$ correlation, but usually the gradient of $\overline{T}$ in the x -direction is much smaller than in the y -direction, fluctuations in T are smaller, so $\overline{ut}$ is expected to be an order of magnitude smaller than $\overline{vt}$. Johnson in reference (1) does show results, however, that indicate $\overline{ut}$ to be about twice the magnitude of $\overline{vt}$. In this thesis $\overline{ut}$ cancels out of the formulas used to calculate $\overline{vt}$. Its order of magnitude compared to $\overline{vt}$ is therefore not determined.

It must be remembered that near the 'edge' of the thermal boundary layer $\overline{ut}$ and $\overline{vt}$ could become comparable; however the methods of measurement are unlikely to detect these quantities with much certainty near the free stream.

The concepts of eddy shear stress coefficient and eddy energy coefficient are defined by:

$$\overline{uv} = -\epsilon_m \frac{\partial \overline{U}}{\partial y} \quad (3)$$

and
$$\overline{vt} = -\epsilon_h \frac{\partial \overline{T}}{\partial y}$$

Class for the given $\bar{y}$ (with boundary values in the sub-layer, no $\bar{y}$ and $\bar{y}$ tend to zero. At the edge of the boundary layer where the gradient of $\bar{y}$ disappears, $\bar{y}$ and $\bar{y}$ tend to zero. In a similar fashion no the average is zero.

In summary, the expected behavior of $\bar{y}$ as a function of $\bar{y}$ should appear as shown in Fig. 6. Theoretical predictions leads now to expect a positive correlation of $\bar{y}$ with $\bar{y}$ as a function of $\bar{y}$ over time.

$$(20) \quad \bar{y} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} (y(t)) dt$$

These results are obtained in Fig. 7.

There is also a gradient of temperature in the x-direction produced a $\bar{y}$ correlation, but usually the gradient of $\bar{y}$ in the x-direction is much smaller than in the y-direction. Theoretical predictions for $\bar{y}$ are smaller, so $\bar{y}$ is expected to be an order of magnitude smaller than $\bar{y}$. However, in reference (1) it was shown that, however, that indicates $\bar{y}$ to be about twice the magnitude of $\bar{y}$. In this case the results are in the same order of magnitude as calculated $\bar{y}$. For order of magnitude comparison to $\bar{y}$ in this case and definition.

It must be recognized that most of the 'edge' of the thermal boundary layer of $\bar{y}$ would be in the x-direction; however, the values of $\bar{y}$ are much smaller than the values of $\bar{y}$ in the y-direction. The values of $\bar{y}$ are much smaller than the values of $\bar{y}$ in the y-direction.

The correlation of $\bar{y}$ with $\bar{y}$ is expected to be very small.

Related to (20) is defined by

$$(21) \quad \bar{y} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^{+T} (y(t)) dt$$

(4)

where $\mathcal{E}_m$ and $\mathcal{E}_h$ are probably different functions of y .

When these definitions are substituted in equations (1) and (2), the result is

$$\frac{\partial}{\partial x} (\bar{P} + \rho \bar{U}^2 + \rho \bar{u}^2) = \frac{\partial}{\partial y} \left[(\mu + \rho \mathcal{E}_m) \frac{\partial \bar{U}}{\partial y} - \bar{U} \bar{V} \right] \quad (25)$$

and

$$\frac{\partial}{\partial x} (\bar{U} \bar{T} + \bar{u} \bar{t}) = \frac{\partial}{\partial y} \left[(\mathcal{L} + \mathcal{E}_h) \frac{\partial \bar{T}}{\partial y} - \bar{V} \bar{T} \right] \quad (26)$$

Solutions to these equations have been obtained for various simple geometries and assumed empirical distributions of pressure, velocity and temperature. In most cases the ratio of $\mathcal{E}_m$ to $\mathcal{E}_h$ is assumed to be constant or some arbitrary function of y . Very few measurements of $\mathcal{E}_m$ and $\mathcal{E}_h$ are available.

(4)

where $\bar{u}$ and $\bar{v}$ are the mean values of u and v respectively, and $\bar{u}^2$ and $\bar{v}^2$ are the mean values of u^2 and v^2 respectively. The mean values of u and v are given by (1) and (2), and the mean values of u^2 and v^2 are given by (3) and (4).

It is now required to find the mean values of u^2 and v^2 in terms of the mean values of u and v .

$$(5) \quad \left[\bar{u}^2 - \frac{\bar{u}^2}{\bar{v}^2} (\bar{u}^2 + \bar{v}^2) \right] \frac{6}{\bar{v}^2} = (\bar{u}^2 + \bar{v}^2) \frac{6}{\bar{v}^2}$$

and

$$(6) \quad \left[\bar{v}^2 - \frac{\bar{v}^2}{\bar{u}^2} (\bar{u}^2 + \bar{v}^2) \right] \frac{6}{\bar{u}^2} = (\bar{u}^2 + \bar{v}^2) \frac{6}{\bar{u}^2}$$

Equations (5) and (6) have been obtained by dividing the

equations (1) and (2) by the mean values of u and v respectively, and

multiplying the results by the mean values of u^2 and v^2 respectively. It

will be seen that the results are identical with the results obtained by

dividing the equations (1) and (2) by the mean values of u and v respectively, and

multiplying the results by the mean values of u^2 and v^2 respectively. It

will be seen that the results are identical with the results obtained by

dividing the equations (1) and (2) by the mean values of u and v respectively, and

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will be seen that the results are identical with the results obtained by

dividing the equations (1) and (2) by the mean values of u and v respectively, and

multiplying the results by the mean values of u^2 and v^2 respectively. It

Appendix C*

HOT WIRE ANEMOMETER THEORY

A. Steady Flow.

Hot-wire theory has been well developed for various ideal cases. Most of the theory is based on King's equation which appears in various forms such as

$$\frac{H}{k(T_w - T_e)} = C_3 + C_4 \sqrt{N_{Pr} N_{Re}} \quad (27)$$

or alternatively

$$H = (T_w - T_e) \left[C_3 k + C_4 \sqrt{N_{Pr}} \sqrt{\frac{k^2 \rho}{\mu}} \sqrt{v_p D} \right] \quad (28)$$

For a gas, ρ is proportional to P/T_g , and for air $k^2/\mu T_g$ is approximately independent of pressure and temperature such that

$$H = (T_w - T_e) F(PV_p) \quad (29)$$

where

$$F(PV_p) = \left[0.58 + 6.6 \sqrt{P^* v_p^* D^*} \right] \quad (30)$$

The Flow Corporation, Cambridge, Massachusetts supplies hot-wire anemometer equipment which makes use of several interesting techniques. First, the equipment is of the 'constant current' type. Second, all measurements are made with the hot wire at a selected resistance ratio above the cold wire resistance. Third, measurements of unknown signals from the wire are made relative to a known square-wave signal applied to the wire from an internal circuit.

A summary of the principles of wire theory will give adequate under-

* This appendix is based primarily on references (3) and (12).

1990-1991

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Of the chimney is located on the 11th floor of the building. The chimney is located on the 11th floor of the building. The chimney is located on the 11th floor of the building.

$$(12) \quad \sqrt{a^2 x^2 + b^2} + x^2 = \frac{b}{(x - \sqrt{a^2 x^2 + b^2})}$$

we are not

$$(iii) \quad \left[\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} \sqrt{\frac{1}{2} + \frac{1}{2}} \right] (e^{i\pi/4} - e^{i3\pi/4}) = 0$$

That a box is proportional to $V^{1/3}$, and the size of the box is proportional to $V^{1/3}$.

(cont.)

	Cumulative percentage (%)	no.	Age in years	sex
0-10	86.7	19	10	M
10-20	93.3	21	20	F
20-30	96.7	22	30	F
30-40	98.3	23	40	F
40-50	99.2	24	50	F
50-60	99.6	25	60	F
60-70	99.8	26	70	F
70-80	99.9	27	80	F
80-90	100.0	28	90	F
90-100	100.0	29	100	F

$$(12) \quad \left[\sqrt{1.1 + 02.0} \right] = (9.99)$$

The five categories, including, instruments supplied for use in
computer equipment found copies and of several interesting specimens.
First, the equipment is of the 'compact' design. Second, all
specimens are made with the use of a selected minimum ratio
above the call area. Third, specimens of various sizes
from the size and weight relative to a standard specimen applied to
the size of the instrument.

* This material is based in part on (X) and (XII).

standing of wire operation.

For a hypothetical fluctuating flow, the power input to the wire balances the power carried away by the stream,

$$I^2 R = H L = (T_w - T_e) L F(PV_p) \quad (31)$$

Also, neglecting second order terms,

$$R = R_0 \left[1 + \alpha_0 (T_w - T_0) \right] \quad (32)$$

where R_0 , α_0 , and T_0 are arbitrary corresponding quantities. These can be chosen as R_e , α_e and T_e at the 'cold' temperature of the wire assumed to be in equilibrium with the stream ambient temperature. Therefore

$$R = R_e \left[1 + \alpha_e (T_w - T_e) \right] \quad (33)$$

Solving (33) for $(T_w - T_e)$ and making use of $\mathcal{F} = R/R_e$, equation (31) becomes

$$I^2 \left(1 - \frac{1}{\mathcal{F}} \right) = \frac{L F(PV_p)}{\alpha_e R_e} \quad (34)$$

In steady state, therefore, a measurement of I will give the PV_p product since all other quantities are fixed. In practice for a particular wire at low velocities, a calibration of I vs V_p is made assuming F is nearly constant. Because it usually results in a straight line, the calibration is usually plotted as I^2 vs $\sqrt{V_p}$ giving ^a graph that may be represented by an equation of the form

$$I^2 = C_1 + C_2 \sqrt{V_p} \quad (35)$$

The derivative with respect to velocity is

$$\frac{dI}{dV_p} = \frac{C_2}{4 I \sqrt{V_p}} \quad (36)$$

In Flow Corporation literature, the quantity dI/dV_p is denoted by I' .

3. Turbulent Flow.

When the stream is turbulent, no steady state exists and the operation of the wire must be analyzed from a time dependent approach. The wire stores heat. Since it cannot change heat storage instantaneously, a power loss balance for the wire is

$$c A L \frac{\partial T_w}{\partial \tau} = I^2 R - \frac{L (\mathcal{F} - 1)}{\mathcal{L}_e} F(PV_p) \quad (37)$$

From equation (33)

$$\frac{\partial T_w}{\partial \tau} = \frac{1}{\mathcal{L}_e R_e} \frac{\partial R}{\partial \tau} \quad (38)$$

$$\text{Take } R = \bar{R} + r \quad (39)$$

$$\text{and } F(PV_p) = F(\bar{P}\bar{V}_p + \bar{P}v_p) \quad (40)$$

where also

$$F(\bar{P}\bar{V}_p + \bar{P}v_p) = F(\bar{P}\bar{V}_p) + \left. \frac{\partial F}{\partial v_p} \right|_p v_p = \bar{F}(PV_p) + F' v_p \quad (41)$$

Substituting these in equation (37), expanding and subtracting the steady state equation (31) leaves

$$c A L \frac{\partial r}{\partial \tau} = \mathcal{L}_e R_e I^2 r - L (\bar{R} - R_e) F' v_p - L \bar{F} r \quad (42)$$

Since the voltage across the wire is

$$e_w = I r \quad (43)$$

equation (42) becomes

$$c A L \frac{d e_w}{d \tau} = [\mathcal{L}_e R_e I^2 - L \bar{F}] e_w - L (\bar{R} - R_e) I F' v_p \quad (44)$$

Solving for v_p :

$$v_p = - \left[\frac{L \bar{F} - \mathcal{L}_e R_e I^2}{L (\bar{R} - R_e) I F'} e_w + \frac{c A L}{L (\bar{R} - R_e) I F'} \frac{d e_w}{d \tau} \right] \quad (45)$$

Thus, the fluctuating velocity component, v_p , is a function of the voltage and the time derivative of the voltage induced in the wire by the velocity. The Flow Corporation HWS performs this operation such that the

output voltage is

$$e_s = J_1 e_w + J_2 \frac{d e_w}{d \tau} \quad (46)$$

J_2 is a constant but J_1 is adjustable so that

$$J_1 = J_2 \left(\frac{L \bar{F} - \mathcal{L}_e R_e I^2}{c A L} \right) \quad (47)$$

Solving (45) for e_w or $d e_w / d \tau$ and substituting in (46)

$$e_s = - \left[\frac{J_1 L (\bar{R} - R_e) I F'}{L \bar{F} - \mathcal{L}_e R_e I^2} \right] v_p = - \left[\frac{J_2 I (\bar{R} - R_e) F'}{c A} \right] v_p \quad (48)$$

if equation (47) is true. The adjustment of the ratio of J_1 / J_2 is made by observing the amplifier output on an oscilloscope when a square-wave current is fed to the wire. The process of 'compensating' the amplifier is detailed in Appendix D. At the point of proper compensation with the wire in still air, the wire response to the square-wave current is

$$e_c = \left[\frac{2 J_1 \bar{I}^2 \bar{R} R_e \mathcal{L}_e}{L \bar{F} - \bar{I}^2 \mathcal{L}_e R_e} \right] i_s = \left[\frac{2 J_2 \bar{I}^2 \bar{R} R_e \mathcal{L}_e}{c A L} \right] i_s \quad (49)$$

$$\text{where } I = \bar{I} + i_s \quad (50)$$

The ratio of e_s to e_c is

$$\frac{e_s}{e_c} = \left[\frac{L (\bar{R} - R_e) I F'}{2 \bar{I}^2 \bar{R} R_e \mathcal{L}_e i_s} \right] v_p \quad (51)$$

$$\text{Since } F' = \frac{2 \mathcal{L}_e R_e}{L} \left(\frac{\mathcal{F}}{\mathcal{F}-1} \right) \bar{I} I' \quad (52)$$

$$\text{then } \frac{e_s}{e_c} = \frac{I'}{i_s} v_p, \text{ or } v_p = \frac{i_s}{I'} \frac{e_s}{e_c} \quad (53)$$

(16)

$$\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) = 1$$

(17)

$$\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) = 1$$

Solving (16) for γ or γ^2 and substituting in (17)

$$(18) \quad \left[\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) \right] = 1$$

is equation (18) is given. The adjustment of the ratio γ is such

by observing the behavior of the oscillation when a square-wave

current is fed on the wire. The process of "correcting" the oscillator is

described in equation (18). At the point of proper compensation with the wire

in unity the wire response to the square-wave current is

$$(19) \quad \left[\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) \right] = 1$$

(20)

$$\gamma + \bar{\gamma} = 1$$

the ratio of γ to $\bar{\gamma}$ is

(21)

$$\left[\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) \right] = 1$$

(22)

$$\gamma + \bar{\gamma} = 1$$

(23)

$$\frac{1}{\gamma} \left(\frac{1}{\gamma} + \frac{1}{\gamma} \right) = 1$$

It must be realized that in the derivation of all equations, fluctuating components are small compared with the average values. This allows neglecting second-order products of the fluctuating components as being immeasurably small. Even when this is not strictly true, the method of normalizing represented by equation (51) compensates for some deviation.

C. Turbulent Flow with Temperature Fluctuations.

A fluid in turbulent flow past a heated surface will become heated in some fluctuating manner. The effect of varying fluid temperature may be regarded as a change in the 'cold' resistance of the wire, R_c , such that

$$R_g = \bar{R}_g + r_g \quad (54)$$

$$= R_c \left[1 + \alpha_e (T_g - T_a) \right] \quad (55)$$

Combining these with the power balance equations in the presence of velocity fluctuations, plus the relations

$$\begin{aligned} e_w &= I r \\ e_s &= J_1 \left(e_w + \frac{c A L}{L \bar{F} - \bar{I}^2 \alpha_e R_e} \frac{d e_w}{d z} \right) \\ e_c &= \left(\frac{2 J_1 \bar{I}^2 \bar{R} R_e \alpha_e}{L \bar{F} - \bar{I}^2 R_e \alpha_e} \right) i_s \\ F' &= \frac{2 \bar{I} I' \alpha_e R_e}{L} \frac{\bar{R}}{\bar{R} - \bar{R}_g}, \quad \text{and} \\ F(PV_p) &= \frac{\alpha_e R_e \bar{I}^2 \bar{R}}{L (\bar{R} - \bar{R}_g)} \end{aligned}$$

gives

$$2 i_s \frac{e_s}{e_c} + 2 I' v_p - \frac{\bar{I}}{\bar{R} - \bar{R}_g} r_g = 0 \quad (56)$$

It must be pointed out in the derivation of all equations, the
 existing components are still regarded with the average values. This al-
 lows regarding known-order products of the fluctuating components as
 being insignificantly small. Now, when this is not strictly true, the
 method of perturbation representation by equation (11) requires the use
 of iteration.

2. Perturbation with respect to Temperature Fluctuations.

A field in equilibrium with a heated surface will become heated
 in some characteristic manner. The effect of varying field dependence may
 be regarded as a change in the 'coil' resistance of the wire, R_{co} , such

$$(2a) \quad R_{co} = R_0 + \Delta R_{co}$$

$$(2b) \quad R_{co} = R_0 \left[1 + \alpha \left(T - T_0 \right) \right]$$

Combined these with the power balance equation in the presence of

velocity fluctuations, gives the following

$$\begin{aligned} \frac{d^2 \bar{T}}{dt^2} + \frac{d \bar{T}}{dt} + \frac{R_0}{L} \bar{T} &= \frac{R_0}{L} T_0 \\ \frac{d^2 \Delta T}{dt^2} + \frac{d \Delta T}{dt} + \frac{R_0}{L} \Delta T &= \frac{R_0}{L} \Delta T_0 \\ \frac{d^2 \bar{T}}{dt^2} + \frac{d \bar{T}}{dt} + \frac{R_0}{L} \bar{T} &= \frac{R_0}{L} T_0 \\ \frac{d^2 \Delta T}{dt^2} + \frac{d \Delta T}{dt} + \frac{R_0}{L} \Delta T &= \frac{R_0}{L} \Delta T_0 \end{aligned}$$

$$(2c) \quad \bar{T} = T_0 + \Delta T$$

Noting that $T_g = \bar{T}_g + t$ (57)

$$\bar{R}_g = R_e \left[1 + \alpha_e (T_g - T_e) \right] \quad (55)$$

and

$$r_g = \alpha_e R_e t \quad (58)$$

leads to

$$2 I_s \frac{e_s}{e_c} + 2 I' v_p - \frac{\alpha_e \bar{I}}{\gamma - [1 + \alpha_e (\bar{T}_g - T_e)]} t = 0 \quad (59)$$

e_s and e_c have been taken as the outputs from an ideal amplifier.

An actual amplifier will have an output of $(e_s + e_n)$ and $(e_c + e_n)$. When this output is fed to a true RMS voltmeter, the latter will have a reading of

$$M_{sn} = K_1 \sqrt{\lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (e_s + e_n)^2 d\tau} \quad (60)$$

$$\text{or } M_{cn} = K_1 \sqrt{\lim_{\tau \rightarrow \infty} \frac{1}{2\tau} \int_{-\tau}^{+\tau} (e_c + e_n)^2 d\tau} \quad (61)$$

But the time average of the voltage is $\overline{(e_s + e_n)^2} = \overline{e_s^2} + \overline{e_n^2}$

since the time average of $\overline{e_s e_n} = 0$. That is, there is no correlation between noise and signal.

Therefore,

$$M_{sn} = K_1 \sqrt{\overline{e_s^2} + \overline{e_n^2}} \quad (62)$$

and

$$M_n = K_1 \sqrt{\overline{e_n^2}} \quad (63)$$

$$\text{It follows that } \sqrt{M_{sn}^2 - M_n^2} = K_1 \sqrt{\overline{e_s^2}} \quad (64)$$

the RMS value of e_s . For the square-wave

$$\sqrt{M_{scn}^2 - M_{sn}^2} = K_1 \sqrt{\overline{e_c^2}} = K_1 e_c. \quad (65)$$

Squaring equation (59) and making use of the meter relations gives

$$(27) \quad \bar{z} + \bar{z} = \bar{z} \quad \text{writing time}$$

$$(28) \quad \left[(z - \bar{z})^2 + 1 \right] z = \bar{z}$$

$$(29) \quad z + \bar{z} = z$$

$$(30) \quad z = \frac{\bar{z} z}{\left[(z - \bar{z})^2 + 1 \right] - z} + \frac{z}{z} = \frac{\bar{z} z}{\left[(z - \bar{z})^2 + 1 \right] - z} + 1$$

and z have been taken as the output from an ideal amplifier. The second amplifier will have an output of $(z + \bar{z})$ and $(z + \bar{z})$ and this output is fed to a third amplifier, the output will have a voltage

$$(31) \quad z = \frac{1}{\sqrt{1 + \frac{1}{z^2}}} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z}^{+z} (z + \bar{z})^2 dz$$

$$(32) \quad z = \frac{1}{\sqrt{1 + \frac{1}{z^2}}} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z}^{+z} (z + \bar{z})^2 dz$$

but the time average of the voltage is $\bar{z} + \bar{z} = \bar{z} + \bar{z}$ since the time average of $z = 0$. The time average is the same between pairs and pairs.

$$(33) \quad \bar{z} = \frac{1}{\sqrt{1 + \frac{1}{z^2}}} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z}^{+z} (z + \bar{z})^2 dz$$

$$(34) \quad \bar{z} = \frac{1}{\sqrt{1 + \frac{1}{z^2}}} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z}^{+z} (z + \bar{z})^2 dz$$

$$(35) \quad \bar{z} = \frac{1}{\sqrt{1 + \frac{1}{z^2}}} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z}^{+z} (z + \bar{z})^2 dz$$

$$(36) \quad \bar{z} = \frac{1}{\sqrt{1 + \frac{1}{z^2}}} \lim_{z \rightarrow \infty} \frac{1}{z} \int_{-z}^{+z} (z + \bar{z})^2 dz$$

integrating equation (36) and taking use of the other relations gives

$$\begin{aligned}
 (2 I_s)^2 \left[\frac{M_{scn}^2 - M_{si}^2}{M_{scn}^2 - M_{sn}^2} \right] &= (2 I')^2 \frac{v_p^2}{t^2} - \left\{ \frac{4 \alpha_e \bar{I} I'}{\sigma - [1 + \alpha_e (\bar{T}_g - T_e)]} \right\} \frac{v_p}{t} \\
 &+ \left\{ \frac{\alpha_e \bar{I}}{\sigma - [1 + \alpha_e (\bar{T}_g - T_e)]} \right\}^2 \frac{1}{t^2} \quad (66)
 \end{aligned}$$

A set of simultaneous equations can be obtained by measuring all quantities at several resistance ratios. For more than three resistance ratios the unknowns are overdetermined and the method of averages or least squares can be used to evaluate the unknowns.

D. Resolution of Velocity Components.

The foregoing analysis assumed that the velocity was perpendicular to the axis of the wire. Within 70° or more from the perpendicular, the cooling effect on the wire varies as the cosine of the angle, that is $V_\theta = V_p \cos \theta$. A wire at 45° to the average stream direction has an average effective velocity $1/\sqrt{2}$ times the stream velocity. This applies equally to the steady value and the fluctuating value of the velocity.

Consider only the fluctuating components in the turbulent stream, because the amplifier filters out all DC wire response components. For two wires at $\pm 45^\circ$ to the stream,

$$v_a = \frac{u + v}{\sqrt{2}} \quad (67)$$

$$v_b = \frac{u - v}{\sqrt{2}} \quad (68)$$

Setting $v_a = v_p$, then $v_b = v_p$, equation (66) becomes

$$(66) \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left\{ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \right\} + \left\{ \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right] \right\} = \left[\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right]^2$$

A set of simultaneous equations can be obtained by assuming all quantities as general variables rather than as specific values. The ratios the unknowns are determined and the system of equations of least squares can be used to evaluate the unknowns.

5. Evaluation of Velocity Components

The foregoing analysis assumes that the velocity was perpendicular to the axis of the wire. Figure 10 shows that the perpendicular, the cooling effect on the wire varies as the cosine of the angle, that is, $\cos \theta = \frac{v}{c}$. If θ is the angle between the direction of the average relative velocity $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ and the direction of the wire, then the ratio of the steady value and the fluctuating value of the voltage is $\cos \theta$.

Consider only the fluctuating component in the induced voltage, because the amplifier filters out all DC voltage components. The two wires are $\pm 45^\circ$ to the current.

$$(67) \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$(68) \quad \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Because $v = v_x$, then $v_y = v_z$, equation (68) becomes

$$\begin{aligned}
 (2 i_s)_a^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_a &= 2 I_a'^2 (\overline{u^2} + 2\overline{uv} + \overline{v^2}) \\
 &\quad - \frac{2\sqrt{2} \mathcal{L}_e I_a' \bar{I}_a}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (\overline{ut} + \overline{vt}) \\
 &\quad + \left\{ \frac{\mathcal{L}_e \bar{I}_a}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \right\}^2 \overline{t^2}
 \end{aligned} \tag{69}$$

and

$$\begin{aligned}
 (2 i_s)_b^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_b &= 2 I_b'^2 (\overline{u^2} - 2\overline{uv} + \overline{v^2}) \\
 &\quad - \frac{2\sqrt{2} \mathcal{L}_e I_b' \bar{I}_b}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (\overline{ut} - \overline{vt}) \\
 &\quad + \left\{ \frac{\mathcal{L}_e \bar{I}_b}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \right\}^2 \overline{t^2}
 \end{aligned} \tag{70}$$

Subtracting (69) from (70):

$$\begin{aligned}
 4 \left\{ i_{sb}^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_b - i_{sa}^2 \left[\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right]_a \right\} &= \\
 2 (I_b'^2 - I_a'^2) (\overline{u^2} + \overline{v^2}) & \\
 - 4 (I_b'^2 + I_a'^2) \overline{uv} & \\
 - \frac{2\sqrt{2} \mathcal{L}_e}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (I_b' \bar{I}_b - I_a' \bar{I}_a) \overline{ut} & \\
 + \frac{2\sqrt{2} \mathcal{L}_e}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} (I_b' \bar{I}_b + I_a' \bar{I}_a) \overline{vt} & \\
 + \left\{ \frac{\mathcal{L}_e}{\gamma' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \right\}^2 (\bar{I}_b^2 - \bar{I}_a^2) \overline{t^2} &
 \end{aligned} \tag{71}$$

$$(40) \quad \frac{(\bar{y} + \sqrt{b} + \bar{y})^2}{2} = \left[\frac{\bar{y}^2 + 2\bar{y}\sqrt{b} + b}{2} \right] \quad (40)$$

$$(b) \quad \frac{(\bar{X}_1 + \bar{X}_2 - \bar{X}_0) \frac{1}{2} \sqrt{b}}{\left[\frac{\bar{X}_1^2 - \bar{X}_2^2}{2\bar{X}_1 - \bar{X}_2} - \frac{\bar{X}_2^2}{2} \right] - \gamma} = \left[\frac{\frac{\bar{X}_1^2 - \bar{X}_2^2}{2\bar{X}_1 - \bar{X}_2} - \frac{\bar{X}_2^2}{2}}{\bar{X}_1^2 - \bar{X}_2^2} \right] \frac{1}{2} \sqrt{b}$$

$$= \left\{ \frac{\frac{1}{\alpha} - \frac{1}{\beta}}{\frac{1}{\alpha} - \frac{1}{\beta}} \right\} - \left\{ \frac{\frac{1}{\alpha} - \frac{1}{\beta}}{\frac{1}{\alpha} - \frac{1}{\beta}} \right\}$$

$$\begin{aligned} & (\overline{v}_0 + \overline{v}_1)(\overline{z}_1 - \overline{z}_2) + \\ & \overline{w}(\overline{z}_2 + \overline{z}_3) + \\ & \overline{w}(\overline{z}_1 - \overline{z}_2) \frac{2\sqrt{x}}{[(\overline{z}_1 - \overline{z}_2)_{2+1}]^{-\gamma}} + \\ & \overline{w}(\overline{z}_2 + \overline{z}_3) \frac{2\sqrt{x}}{[(\overline{z}_1 - \overline{z}_2)_{2+1}]^{-\gamma}} + \\ (17) \quad & \overline{z}_1(\overline{z}_1 - \overline{z}_2) \left\{ \frac{2}{[(\overline{z}_1 - \overline{z}_2)_{2+1}]^{-\gamma}} \right\} + \end{aligned}$$

Fortunately, the two most important quantities of interest in the boundary layer analysis are the dominant terms in this equation. It reduces to

$$(2 I_s)^2 \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right] = - 8 I'^2 \overline{uv} + \frac{2\sqrt{2} (\bar{I}_a + \bar{I}_b) I' \mathcal{L}_e}{\delta' - [1 + \mathcal{L}_e (\bar{T}_g - T_e)]} \overline{vt} \quad (72)$$

for two ideally matched wires or a single wire turned successively to the two orientations of $\pm 45^\circ$ to the stream.

APPENDIX D

EXPERIMENTAL EQUIPMENT AND FACILITIES

(a) Wind Tunnel

The experiments were conducted in a tunnel whose test area has a one foot square cross section. Atmospheric air was drawn through the tunnel by an exhaust fan powered by a one quarter horsepower AC motor. The flow rate through the test section was controlled by a damper in the exhaust line. The maximum velocity attained was approximately twenty-six feet per second. The velocities were measured by a pilot tube probe located on the axis of the tunnel three feet upstream from the test section. A micromanometer was connected to this probe.

The boundary layer was developed over a flat plate which consisted of a twenty-nine inch unheated starting length and a twelve inch heated section. The heating element provided approximately constant heat transfer per unit area. The blunt leading edge of the unheated plate tripped the momentum boundary layer. As shown in Appendix E, the thickness of the momentum and thermal boundary layers at the test station were estimated to be 1.32" and 0.48" respectively.

(b) True RMS Millivoltmeter

An instrument that measures the true root-mean-square voltage of a random input signal was required. There are two commercial meters available which perform this function. They are the Ballantine Laboratories, Inc., model 320 and Flow Corporation, model TM. The principal difference between them is the manner in which the input signal is averaged. The

model 320 uses an amplifier and a squaring circuit of the segmented type. The model TBM employs a unique design thermal element. The principal characteristics of these instruments are:

<u>Characteristic</u>	<u>Model 320</u>	<u>TBM</u>
meter range (rms volts)	0.0001-320	0.0001-250
frequency response (cycles per second)	15-150,000	2-250,000
linearity over frequency range	$\pm 3\%$	$\pm 2\%$
time constant, including meter movements (seconds)	2.25	15
peak factor $\frac{P}{E}$ ratio of period to pulse width	—	10
crest factor $\frac{P}{E}$ ratio of peak to rms amplitude	5	—

Due to the presence of many low frequency components in the turbulence velocities being measured, a relatively long averaging period is required for accurate results. The problems encountered with these meters are presented in Section IV.

(c) Probes

Both a single wire and a V-wire probe were used in this series of experiments. Each had tungsten wires of 0.00015" diameter with sensitive lengths of 0.040". These probes are shown in figure 4.

The single wire probe was used to simulate an X-wire probe as noted in the discussion of results. The best results were obtained in this manner. The V-wire probe had a 90° included angle, and the sensitive segments of the wires were located as close as possible to the apex. The

model 320 uses an oscillator and a recording circuit of the standard type. The model 320 employs a unique design circuit element. The following characteristics of these instruments are:

Model 320	Model 320	Model 320
0.0001-100	0.0001-100	0.0001-100
2-100,000	2-100,000	2-100,000
$\pm 1\%$	$\pm 1\%$	$\pm 1\%$
12	12	12
10	10	10
—	—	—

Due to the presence of water in the frequency components in the analysis velocity being measured, a relatively long averaging period is required for accurate results. The frequency components with these values are recorded in Section 17.

(c) Probes

Both a single wire and a V-wire probe were used in this section of experiments. Each had frequency range of 0.0001-100,000 cycles per second. These probes are shown in Figure 4.

The single wire probe was used to obtain an 8-10 cps response noted in the discussion of results. The best results were obtained in this manner. The V-wire probe had a 90° included angle, and the sensitivity of the wire probe located as close as possible to the apex. The

resistance of the wires was matched to less than two percent. The problems previously discussed severely limited the use of this probe.

The probes were mounted on a traversing mechanism with a micrometer adjustment to accurately measure the distance from the heated plate. A calibrated index was used to change the angular orientation of the probe.

A carefully constructed X-wire probe correctly oriented to the free stream velocity vector would satisfy the assumptions made in the derivation of equation (72). It would eliminate the principal problems encountered with the V-wire probe - i.e., space resolution and needle resistance. Such a probe was not constructed for this thesis due to fund limitations.

(d) Anemometer Circuits

There are three principal circuits required to transform the wire signal into the data required by equation (72). These are the resistance bridge, square-wave circuit, and amplifier. They are conveniently packaged in the Flow Corporation HNB unit. No other commercially available anemometer equipment meets the requirements for measuring fluctuation quantities.

The bridge circuit provides a method for accurately measuring the wire resistance. This is the fundamental quantity which is sensitive to velocity and temperature fluctuations. The resistance of the wire at any operating condition is calculated from measurements of current ($\bar{I}$), voltage (C_w) and resistance ratio (γ).

The amplifier section boosts the AC components of the voltage across the hot wire to a readable level. It also takes the time derivative of the wire voltage and adds it to a fraction of the wire voltage. This fraction is selected by adjusting the compensation dial to obtain minimum

testament of the wire was returned to less than the error. The problem
 previously discussed, namely, limited the use of this probe.
 The probe was mounted on a traveling mechanism with a stationary
 adjustment to accurately measure the distance from the probe plane. A
 calibrated index was used to change the angular relationship of the probe.
 A carefully constructed frame was customarily mounted to the probe
 since velocity vector would indicate the suspension point in the distance
 of equation (12). It is also possible to produce a similar problem statement
 with the 9-wire probe - i.e., speed resolution and transfer resistance. Such
 a probe was not constructed for this thesis due to time limitations.

(d) Instrumentation

There are three principal elements required to transform the wire
 signal into the data required by equation (12). These are the resistance
 bridge, square-wave circuit, and amplifier. They are customarily used
 again in the wire measurement and wire. In other commercially available
 measurement equipment such as the tape recorder for measuring fluctuation
 quantities.

The bridge circuit provides a means for accurately measuring the
 wire resistance. This is the fundamental quantity which is sensitive to
 velocity and temperature fluctuations. The resistance of the wire at any
 operating condition is calculated from measurement of current (1), voltage
 (2), and resistance ratio (3).

The amplifier section provides the AC component of the bridge across
 the AC wire to a readable level. It also takes the time derivative of
 the wire voltage and also it is a function of the wire voltage. This
 function is related by adjusting the compensation also to obtain uniform

signal distortion. To properly set the compensation, a square wave is impressed across the bridge circuit and the amplified wire signal is displayed on an oscilloscope. Correct compensation is obtained when the square wave appears linear with a sharp peak at the leading edge. This vertical peak superimposed on a square wave represents the addition of the derivative to the input signal.

Any random input signal to the amplifier may be represented as a Fourier series. The output voltage is therefore a function of frequency. For accurate measurements all of the frequency components should be passed by the amplifier. The HWB has a quoted bandwidth of 2 cps to 100,000 cps. A further error would be introduced by overdriving the amplifier. To prevent this, the attenuator dial was set at '16' which corresponds to 1/16 of the maximum gain.

The square wave circuit provides a reference voltage to the wire for two purposes. First, the square wave is employed to set the amplifier compensation. In addition, it is used to calibrate or normalize the wire voltage that results from stream fluctuations.

One other piece of anemometer equipment, the HWI, was used to a limited extent. It contains a wire current control circuit and a sum-and-difference circuit. It is used in conjunction with the HWB unit and a dual-wire probe. The sum-and-difference of the instantaneous signals from two wires is required to calculate fluctuating velocity components.

(e) Oscilloscope

The output signal of the anemometer amplifier was displayed on an oscilloscope. A Dumont, type 322, dual channel instrument was available in the laboratory. The dual beam presentation will be found particularly useful with an X-wire probe.

signal detection. To prevent the possibility of a false alarm is
represented between the bridge circuit and the amplifier with a gain of
displayed on an oscilloscope. Current compensation is obtained from the
applies wave square wave with a duty cycle at the bridge side. This
vertical wave is superimposed on a square wave represents the position of
the derivative to the input signal.

Any random noise signal in the amplifier may be represented as a
Fourier series. The output voltage is therefore a function of frequency.
For accurate measurements all of the frequency components should be passed
by the amplifier. The BW has a power spectrum of 1 cps to 100,000 cps.
A further noise would be introduced by modulating the amplifier. In
general this, the amplifier will not be of 1% which represents 10
1/10 of the maximum gain.

The output noise circuit provides a reference voltage to the side the
the purpose. Thus, the output may be applied to the amplifier
connection. In addition, it is used to calibrate or normalize the data
voltage that provides good signal-to-noise ratio.

One other aspect of amplifier operation, the BW, was used to a
limited extent. It contains a wide variety of control circuits and a gain
and offset control circuit. It is used in conjunction with the BW and has
a dual-slope process. The pre-amplifier of the instrument is a
from the noise is removed so that the amplifier is only concerned.

(a) Amplifier

The output signal of the instrument amplifier was displayed on an
oscilloscope. A power, 100 W, dual channel instrument was available
in the laboratory. The dual beam operation will be found particularly
useful with an X-ray probe.

APPENDIX E

DATA CALCULATIONS

1. Wire Calibration

Before each wire was calibrated, it was "aged" at a heating current slightly above the maximum operating range. Its velocity response was determined by calibration in the free stream of the tunnel. The wire was oriented perpendicular to the flow and measurements were made of wire current for various resistance ratios. The air velocity was controlled by a damper in the tunnel exhaust line.

A pitot tube with a micromanometer was used to determine the air velocity. A light oil of 0.827 specific gravity filled the manometer tube. All calculations included corrections for atmospheric pressure and temperature in the tunnel; however, since the temperature variations were small, no correction was made for the density changes of the oil. The following equations were employed to calculate velocities in the tunnel:

$$\begin{aligned}\bar{U} &= \sqrt{2gh} = \left[2(g)(h) \frac{0.827 \rho_W}{\rho_A} \right]^{\frac{1}{2}} \\ &= \left[2(32.2 \text{ ft/sec}) \frac{h \text{ inches}}{12 \text{ inches/ft}} \frac{0.827 \times 62.2 \text{ lbs/ft}^3}{\rho_A \text{ lbs/ft}^3} \right]^{\frac{1}{2}} \\ \bar{U} &= 16.6 \left[\frac{h}{\rho_A} \right]^{\frac{1}{2}}\end{aligned}$$

$$\text{where, } \rho_A = \frac{P^{15^\circ\text{C}}_{760\text{mm}} (0.3789) (p \text{ mm Hg})}{(t^\circ\text{C} + 273.20)} = \frac{(0.07657)(0.3789)(p)}{(t + 273.2)}$$

$$\rho_A = \frac{0.02901 p}{(t + 273.2)}$$

Typical calculations for two resistance ratios with data taken on 7/3/58 follows:

$$P_A = 762.15 \text{ mm Hg} ; \quad t_A = 27.5^\circ \text{ C}$$

$$\rho_A = \frac{(0.02901)(762.15)}{(27.5 + 273.2)} = \frac{22.110}{300.7} = 0.07352 \text{ lbs/ft}^3$$

$$\bar{U} = 16.6 \left[\frac{h}{0.07352} \right]^{\frac{1}{2}} = \frac{16.6}{0.2712} \sqrt{h} = 61.2 \sqrt{h}$$

h in	$\sqrt{h}$ (in) ^{1/2}	$\bar{U}$ ft/sec	$\sqrt{\bar{U}}$ (ft/sec) ^{1/2}	I (ma)		I^2 (ma) ²	
				$\gamma=1.2$	$\gamma=1.4$	$\gamma=1.2$	$\gamma=1.4$
0.195	0.442	27.1	5.20	35.06	45.35	1229	2057
0.153	0.391	23.9	4.89	34.43	44.68	1189	1996
0.139	0.373	22.8	4.78	33.90	44.28	1149	1961
0.117	0.342	20.9	4.57	33.80	43.75	1142	1914
0.082	0.286	17.5	4.18	33.18	42.88	1101	1839
0.040	0.200	12.2	3.49	31.35	40.75	983	1661
0.000	0.000	0.0	0.00	23.25	30.32	541	919

This data is plotted in figure 8. The curves follow the general shape predicted by the King equation for hot wires:

$$I^2 = C_1 + C_2 \sqrt{U}$$

2. Determination of $I' \equiv \partial I / \partial U$

The slope of the curve of wire current versus incident velocity (I') is one of the quantities required for the solution of equation 72. Differentiating the King equation, we obtain

$$2 I \cdot \partial I = 0 + C_2 (U)^{\frac{1}{2}} \partial U$$

$$I' \equiv \left(\frac{\partial I}{\partial U} \right)_t = \frac{C_2}{2 I \sqrt{U}}$$

Typical calculations for the following cases are shown in

Table 1.

$$T = 100.0 \text{ sec} \quad \text{or} \quad T = 100.0 \text{ sec}$$

$$P = \frac{(1.000)(100.0)}{(1.000 + 100.0)} = \frac{100.0}{101.0} = 0.9901 \text{ sec}$$

$$\bar{t} = 100.0 \left[\frac{1}{1.000} + \frac{1}{100.0} \right] = 100.0 \left[1.000 + 0.0100 \right] = 101.0 \text{ sec}$$

$\bar{t}$ (sec)	$\frac{\bar{t}}{T}$	$\frac{\bar{t}}{P}$	$\frac{\bar{t}}{T} - \frac{\bar{t}}{P}$	$\frac{\bar{t}}{T} - \frac{\bar{t}}{P}$	$\frac{\bar{t}}{T} - \frac{\bar{t}}{P}$	$\frac{\bar{t}}{T} - \frac{\bar{t}}{P}$
0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.005	0.005	0.005	0.000	0.000	0.000	0.000
0.010	0.010	0.010	0.000	0.000	0.000	0.000
0.015	0.015	0.015	0.000	0.000	0.000	0.000
0.020	0.020	0.020	0.000	0.000	0.000	0.000
0.025	0.025	0.025	0.000	0.000	0.000	0.000
0.030	0.030	0.030	0.000	0.000	0.000	0.000
0.035	0.035	0.035	0.000	0.000	0.000	0.000
0.040	0.040	0.040	0.000	0.000	0.000	0.000
0.045	0.045	0.045	0.000	0.000	0.000	0.000
0.050	0.050	0.050	0.000	0.000	0.000	0.000
0.055	0.055	0.055	0.000	0.000	0.000	0.000
0.060	0.060	0.060	0.000	0.000	0.000	0.000
0.065	0.065	0.065	0.000	0.000	0.000	0.000
0.070	0.070	0.070	0.000	0.000	0.000	0.000
0.075	0.075	0.075	0.000	0.000	0.000	0.000
0.080	0.080	0.080	0.000	0.000	0.000	0.000
0.085	0.085	0.085	0.000	0.000	0.000	0.000
0.090	0.090	0.090	0.000	0.000	0.000	0.000
0.095	0.095	0.095	0.000	0.000	0.000	0.000
0.100	0.100	0.100	0.000	0.000	0.000	0.000

This data is plotted in Figure 1. The curves shown are smooth curves

drawn by the least squares method.

$$\bar{t} = \frac{1}{\frac{1}{T} + \frac{1}{P}}$$

3. Determination of $\bar{t}$ by $\bar{t} = \frac{1}{\frac{1}{T} + \frac{1}{P}}$

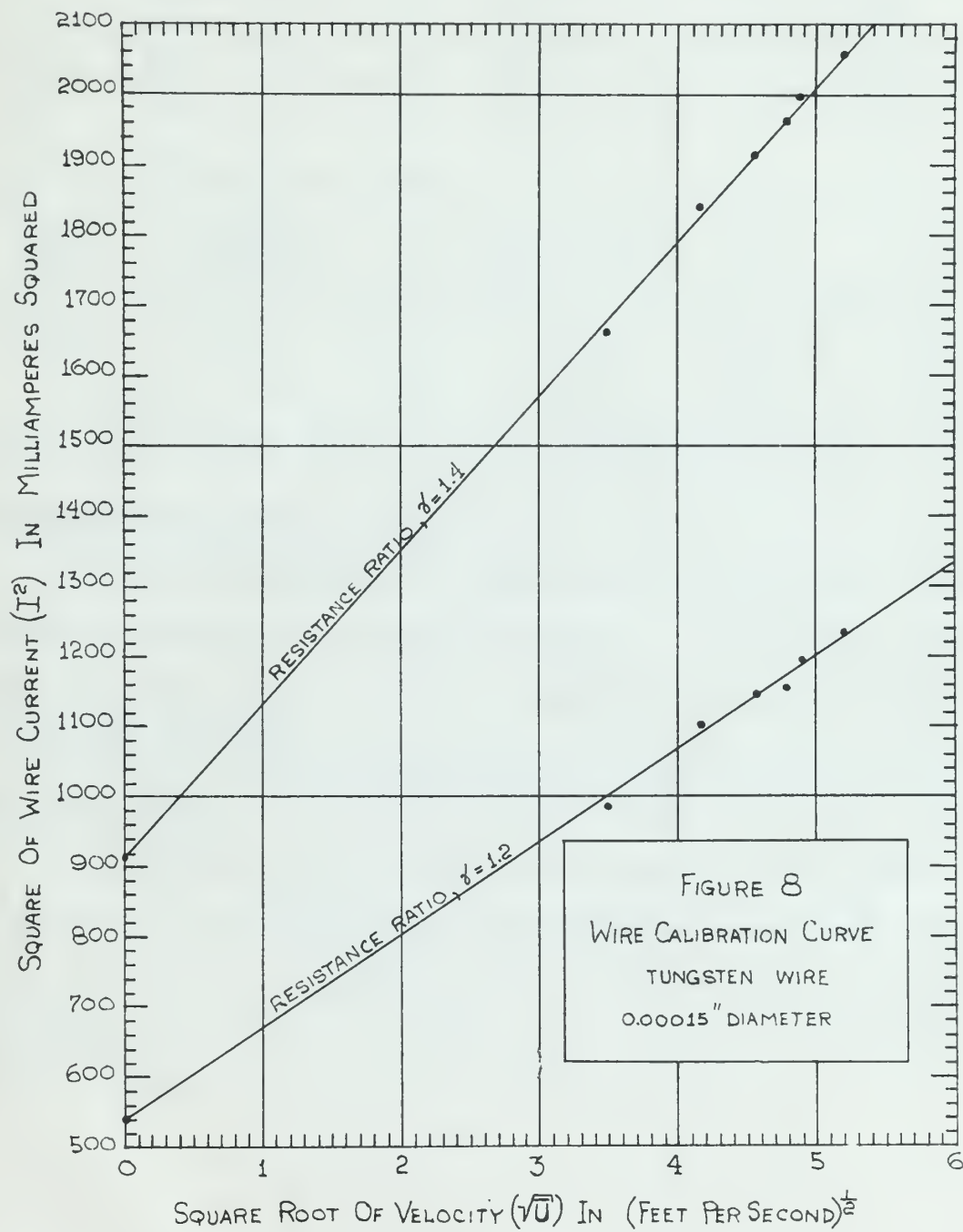
The slope of the curve of $\bar{t}$ versus $\frac{1}{P}$ is constant and is equal to $\frac{1}{T}$.

is not of the parabolic nature for the solution of equation 1. It is

obtaining the first equation, we obtain

$$\bar{t} = \frac{1}{\frac{1}{T} + \frac{1}{P}}$$

$$\bar{t} = \frac{1}{\frac{1}{T} + \frac{1}{P}}$$



where C_2 is the slope of the calibration curves obtained in part 1. above. For the free stream velocity chosen for a particular experimental run, the value of $(\sqrt{U})$ is computed and the corresponding current (I) is read from the calibration curve. With these values, the required slope (I') is calculated. Results are shown for a tunnel velocity of twenty-six feet/second.

U ft/sec	$\sqrt{U}$ (ft/sec) ^{1/2}	γ	C_3 (ma) ² /ft/sec) ^{1/2}	I ma	I' ma/ft/sec
26.0	5.10	1.2	132.6	34.86	0.1865
		1.4	217.4	43.03	0.2366

3. Amplifier Compensation

As discussed in appendix C, a square wave is employed as a reference voltage to determine the anemometer's response to a fluctuating voltage across the wire. The HEB amplifier compensation is set by observing on an oscilloscope the waveform which results when a square wave is impressed on the wire circuit. An undistorted waveform indicates the proper calibration.

To determine the effect of compensation setting on the voltmeter output, measurements were made in the free stream with the hot wire oriented vertically. The values of M_{gn} and M_{gcn} were recorded as the compensation was varied.

where C_2 is the slope of the calibration curves obtained in part I, above. For the time across relay, chosen for a particular experimental run, the value of $(\sqrt{V})$ is computed and the corresponding current (I) is read from the calibration curve. With these values, the required slope (I') is computed. Results are shown for a number of values of frequency in kilocycles.

I' ma/sec	I ma	C_2 (ma) ² /(sec) ²	γ	$\sqrt{V}$ (sec) ^{1/2}	V sec
0.1000	34.40	121.8	1.1	2.10	20.0
0.2500	42.63	217.6	1.4		

3. Miller Compensation

As discussed in Appendix C, a square wave is employed as a reference voltage to determine the instrument's response to a fluctuating voltage across the wire. The only required assumption is that by observing on an oscilloscope the waveforms which result when a square wave is impressed on the wire circuit, an undistorted waveform indicates the proper calibration. To determine the effect of compensation acting on the voltmeter output, measurements were made in the form shown with the not also obtained previously. The values of R_{eq} and R_{int} were recorded as the compensation was varied.

Dial Setting	M_{on} (mv)	M_{ecn} (mv)	M_{sn}^2	M_{ecn}^2	M_n^2	M_s^2	M_c^2	$(\frac{M_s}{M_c})^2$
60	12.8	47.3	164	2230	64	100	2066	0.0484
80	13.3	55.2	176	3040	64	112	2864	0.0391
100	15.6	64.7	242	4176	64	178	3928	0.0454
125	16.2	74.0	262	5460	64	198	5198	0.0381
150	17.2	84.0	296	7050	64	232	6754	0.0344
200	22.2	126	492	15850	64	428	15358	0.0278

This data is plotted in figure 9. The response is approximately linear with no sharp null or peak; hence, proper calibration is difficult. It is estimated that the error introduced in the quantity $(M_s/M_c)^2$ when setting the compensation is of the order of 8.2% when the dial is varied by one dial unit.

4. Square Wave Current

Reference 12 gives the following formula for computing the square current of the HMB:

$$i_s = \frac{3.05}{1 + \frac{(BN)(\gamma)}{400}} = \frac{1220}{400 + (BN)(\gamma)} = \frac{1220}{D}$$

where (BN) is the bridge null setting. It is important to maintain the battery supplying this current near its rated voltage.

Weight (mg)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)	$\frac{W}{W_0}$ (%)
80	12.5	47.3	104	3230	84	100	1000	0.0484
80	12.5	47.3	134	3040	84	112	1084	0.0481
100	12.5	47.3	147	4130	84	134	1058	0.0434
125	12.5	47.3	147	1460	84	146	1194	0.0381
150	17.2	44.9	184	1004	84	172	1134	0.0344
200	21.1	41.1	184	12420	74	184	12120	0.0324

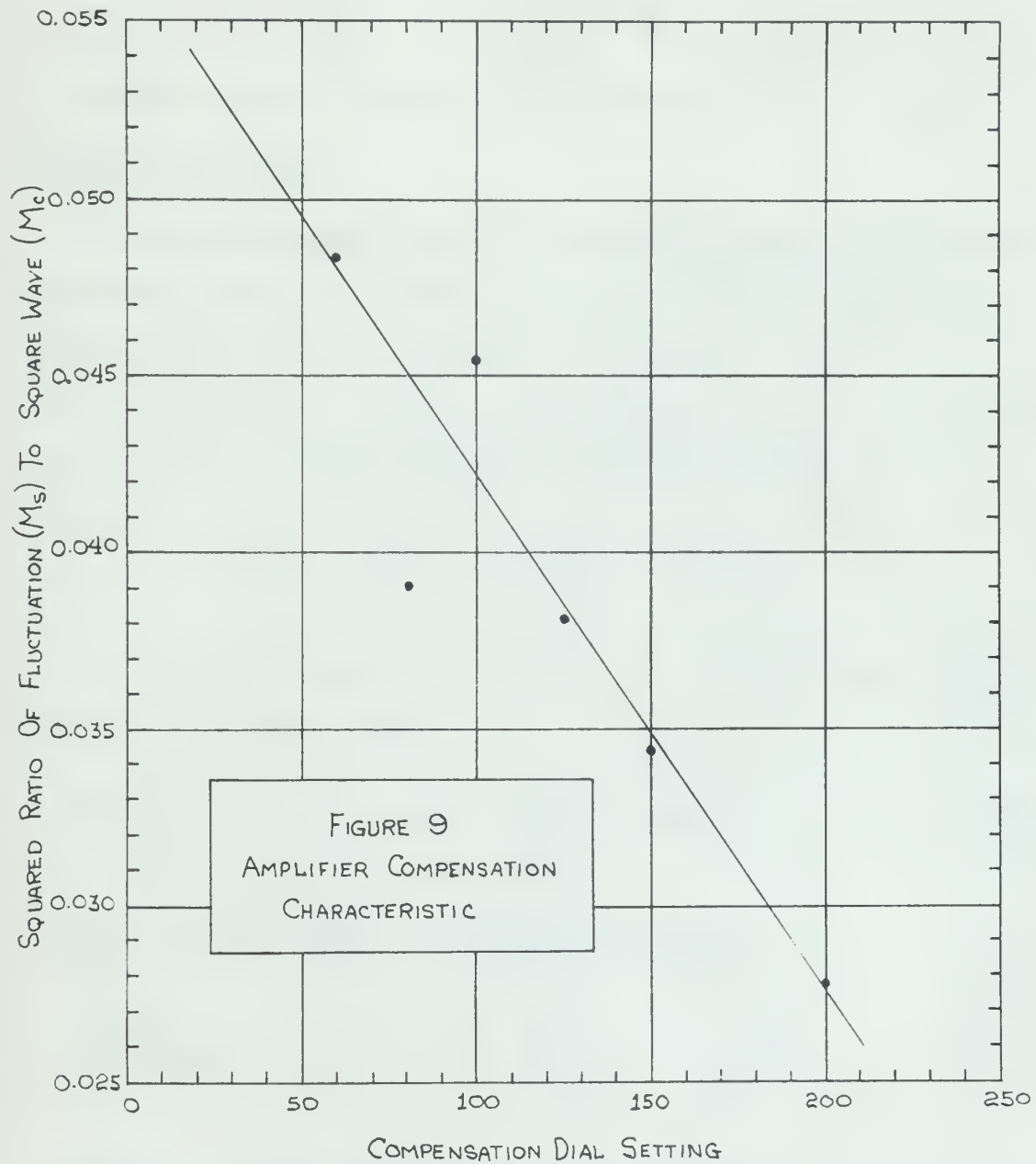
This data is plotted in Figure 6. The response is approximately linear with no sharp fall or peak. Hence, proper calibration is sufficient. It is indicated that the error introduced in the quantity $(\frac{W}{W_0})^2$ when using the composition is of the order of 0.1% when the data is varied by one digit only.

4. BEHAVIOR OF THE SYSTEM

Reference is given to the following formula for computing the system constant of the unit:

$$K = \frac{1.00}{1 + \frac{1.00}{(W_0 + (W_0 - 1))}} = \frac{1.00}{1.00 + (W_0 - 1)} = \frac{1.00}{W_0}$$

where (W_0) is the design unit weight. It is apparent that the system constant is directly proportional to the design weight.



γ	BN	$(\gamma)(BN)$	D	i_s (ma)
1.2	477	572.4	972.4	1.255
1.4	477	667.8	1067.8	1.142

5. Temperature Profile

The pertinent formula for calculating the difference (ΔT) between the temperature (T_g) at a point in the thermal boundary layer and the stream ambient temperature is derived as follows:

$$F(pV) = \frac{I_1^2 R_o \alpha}{L} \left(\frac{R}{R-R_g} \right)^2 = \frac{I_2^2 R_o \alpha}{L} \left(\frac{R}{R-R_e} \right)^2$$

$$\text{from which, } \left(\frac{I_1}{I_2} \right)^2 = \frac{R - R_g}{R - R_e} = \frac{T - \bar{T}_g}{T - T_e}$$

where (I_1) is the wire current for a cold plate and (I_2) is that current for the heated plate.

$$\text{Taking } R = R_g \left[1 + \alpha_g (T - \bar{T}_g) \right] \text{ and rearranging, } T = \bar{T}_g + \left(\frac{\gamma-1}{\alpha_g} \right)$$

$$\text{substituting, } \left(\frac{I_1}{I_2} \right)^2 = \frac{[(\gamma-1)/\alpha_g]}{[(\gamma-1)/\alpha_g] + (\bar{T}_g - T_e)}$$

$$\text{Rearranging, } (\bar{T}_g - T_e) = \frac{\gamma-1}{\alpha_g} \left[1 - \left(\frac{I_1}{I_2} \right)^2 \right];$$

$$\text{taking, } \alpha_g = \frac{\alpha_e}{1 + \alpha_e (\bar{T}_g - T_e)} = \frac{\alpha_e}{1 + \alpha_e (\Delta T)}$$

γ	m	$(\gamma)(m)$	D	λ (cm)
1.2	477	572.4	872.4	1.222
1.4	477	667.8	1067.8	1.142

2. Temperature Profile

The pertinent formula for calculating the difference (ΔT) between

the temperature (T_s) at a point in the central boundary layer and the

stream ambient temperature is derived as follows:

$$\begin{aligned} \theta(\eta) &= \frac{I_1}{I_2} \frac{R}{L} \left(\frac{a}{R-a} \right) \left(\frac{1}{L} \right) = \frac{I_1}{I_2} \frac{R}{L} \left(\frac{a}{R-a} \right) \left(\frac{R}{R-a} \right) \\ \text{from which, } \left(\frac{I_1}{I_2} \right) &= \frac{R-a}{R} = \frac{T - T_s}{T - T_a} \end{aligned}$$

where (I_1) is the wire current for a cold plate and (I_2) is the

current for the heated plate.

Taking $R = R_0 [1 + \alpha (T - T_0)]$ and rearranging, $T = \bar{T}_s + \left(\frac{\gamma-1}{\alpha} \right)$

$$\text{substituting, } \left(\frac{I_1}{I_2} \right)^2 = \frac{[(\gamma-1)\alpha]}{[(\gamma-1)\alpha] + (T_s - T_0)}$$

$$\text{Rearranging, } (\bar{T}_s - T_0) = \frac{\gamma-1}{\alpha} \left[1 - \left(\frac{I_1}{I_2} \right)^2 \right]$$

$$\text{Taking, } \alpha = \frac{1}{1 + \alpha (T_s - T_0)} = \frac{1}{1 + \alpha (\Delta T)}$$

and substituting, it can be shown that

$$(T_g - T_e) = \frac{1}{\alpha_o \left[\frac{1}{(\delta - 1) [1 - (I_1/I_2)^2]} - 1 \right]} \quad (73)$$

For an ambient temperature of 24.8°C , the coefficient (α_o) is computed as follows:

$$\alpha_o = \frac{\alpha_o}{1 + \alpha_o(T_e - T_o)} = \frac{0.00454}{1 + 0.00454(24.8^\circ - 20^\circ)} = 0.00444$$

Since the wire is more sensitive to temperature variations at low currents, the data for the temperature profile was taken at a resistance ratio of 1.1. That data and the values computed with equation 73 are shown in the table below and plotted in figure 10.

y (in)	I_1 (ma)	I_2 (ma)	$(\bar{T}_g - T_e)$ ($^\circ\text{C}$)	$\frac{(\bar{T}_g - T_e)}{T_g - T_e}$ ($^\circ\text{C}$)
0.068"	20.91	22.51	3.13°	0.391
0.118"	21.06	24.05	3.78°	0.472
0.168"	22.41	23.42	1.92°	0.240
0.218"	22.75	23.51	1.45°	0.181
0.268"	23.06	23.66	1.13°	0.141
0.368"	23.35	23.74	0.73°	0.091
0.468"	23.92	23.85	-0.14°	-0.018

The normalized temperature profile is plotted in figure 2 of Section III.

and substituted in eq. (1) to give

$$(1) \quad \frac{1}{\alpha} = \frac{1}{\alpha_0} + \frac{1}{\alpha_0} \left[\frac{(\delta - \alpha_0)}{(\delta - \alpha_0)} \right]$$

For an initial concentration of 36.5% the calculated α is

calculated as follows:

$$\alpha = \frac{1}{1 + \frac{0.0001}{0.0001 + 0.0001}} = 0.5000$$

Since the data for the temperature dependence of α are

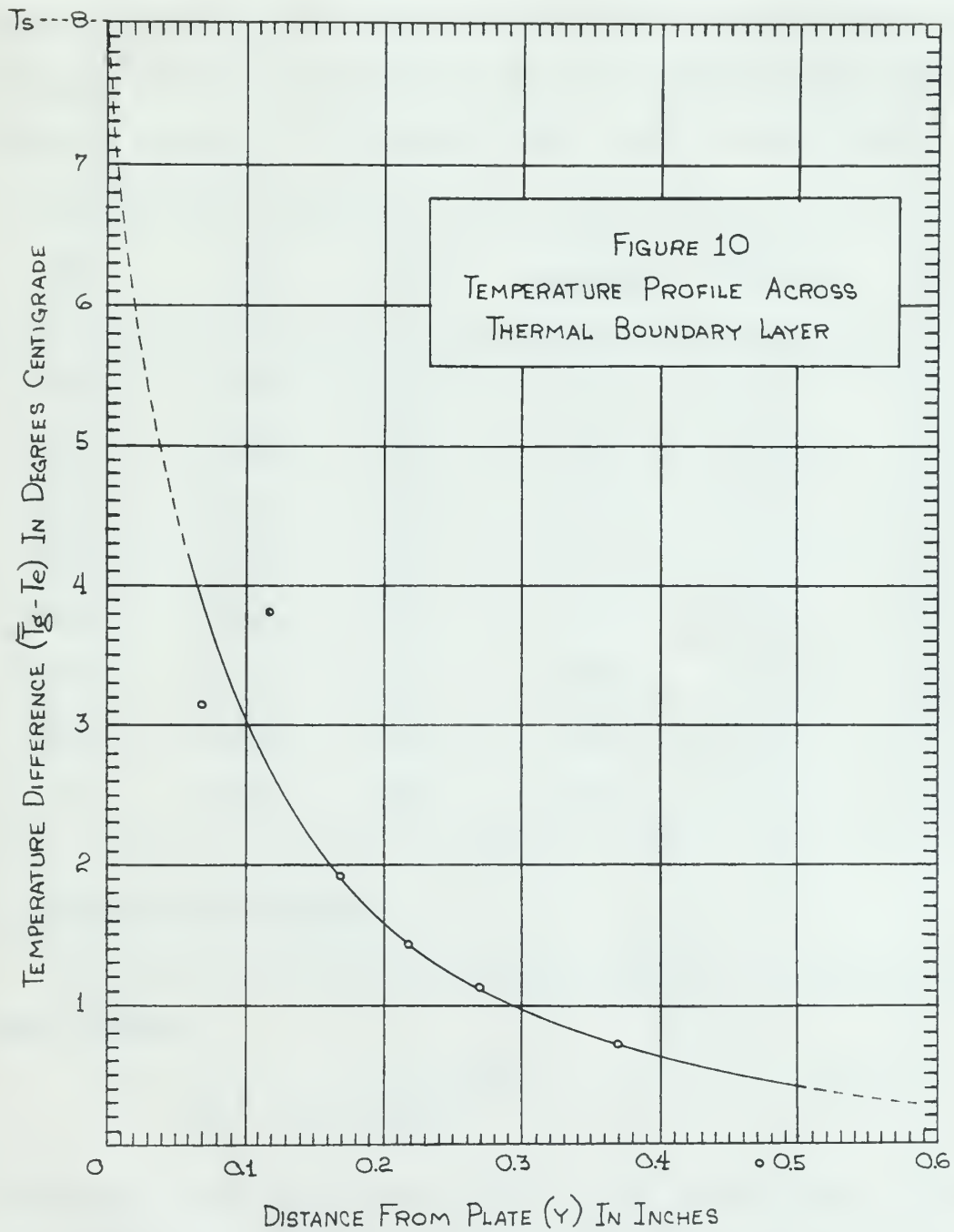
constant, the data for the temperature dependence of α are

shown in Table I. The data for the temperature dependence of α are

shown in Table I and plotted in Figure 1.

$\frac{1}{\alpha} - \frac{1}{\alpha_0}$	$\frac{1}{\alpha} - \frac{1}{\alpha_0}$	$\frac{1}{\alpha} - \frac{1}{\alpha_0}$	$\frac{1}{\alpha} - \frac{1}{\alpha_0}$	$\frac{1}{\alpha} - \frac{1}{\alpha_0}$
0.000	0.000	0.000	0.000	0.000
0.001	0.001	0.001	0.001	0.001
0.002	0.002	0.002	0.002	0.002
0.003	0.003	0.003	0.003	0.003
0.004	0.004	0.004	0.004	0.004
0.005	0.005	0.005	0.005	0.005
0.006	0.006	0.006	0.006	0.006
0.007	0.007	0.007	0.007	0.007
0.008	0.008	0.008	0.008	0.008
0.009	0.009	0.009	0.009	0.009

The calculated temperature dependence of α is shown in Figure 1.



6. Velocity Profile

The velocity profile across the momentum boundary layer is obtained with the plate cold and the wire oriented vertically. The values of current recorded are then used as arguments to enter figure 8 and determine the velocity. The data and results are shown below together with a plot of the profile in figure 11. The normalized plot appears in the RESULTS.

y (in)	I (ma)	I^2 (ma) ²	$\sqrt{\bar{U}}$ (ft/sec) ^{1/2}	$\bar{U}$ (ft/sec)	$\frac{\bar{U}}{U_\infty}$
0.068"	44.00	1936	4.63	21.44	0.758
0.118"	44.35	1967	4.78	22.85	0.807
0.168"	44.48	1978	4.82	23.23	0.821
0.218"	44.68	1996	4.95	24.50	0.866
0.268"	44.78	2005	4.99	24.90	0.880
0.368"	44.98	2023	5.04	25.40	0.898
0.468"	45.12	2036	5.10	26.01	0.920
ϕ	45.70	2088	5.32	28.30	1.000

7. Boundary Layer Thickness

The following equation may be used to predict the momentum boundary layer thickness,

$$\delta_m = 0.37x \left(\frac{v_\infty x}{\nu} \right)^{-1/5}$$

Considering a point twenty-nine inches from the leading edge of the plate and a free stream velocity of twenty-six feet per second, the thickness will be,

6. Velocity Profile

The velocity profile across the channel boundary layer is obtained with the plate coils and the wire stretched vertically. The values of constant resistance are then used as arguments to enter Figure 5 and determine the velocity. The data and results are then plotted together with a plot of the profile in Figure 11. The theoretical plot appears in the results.

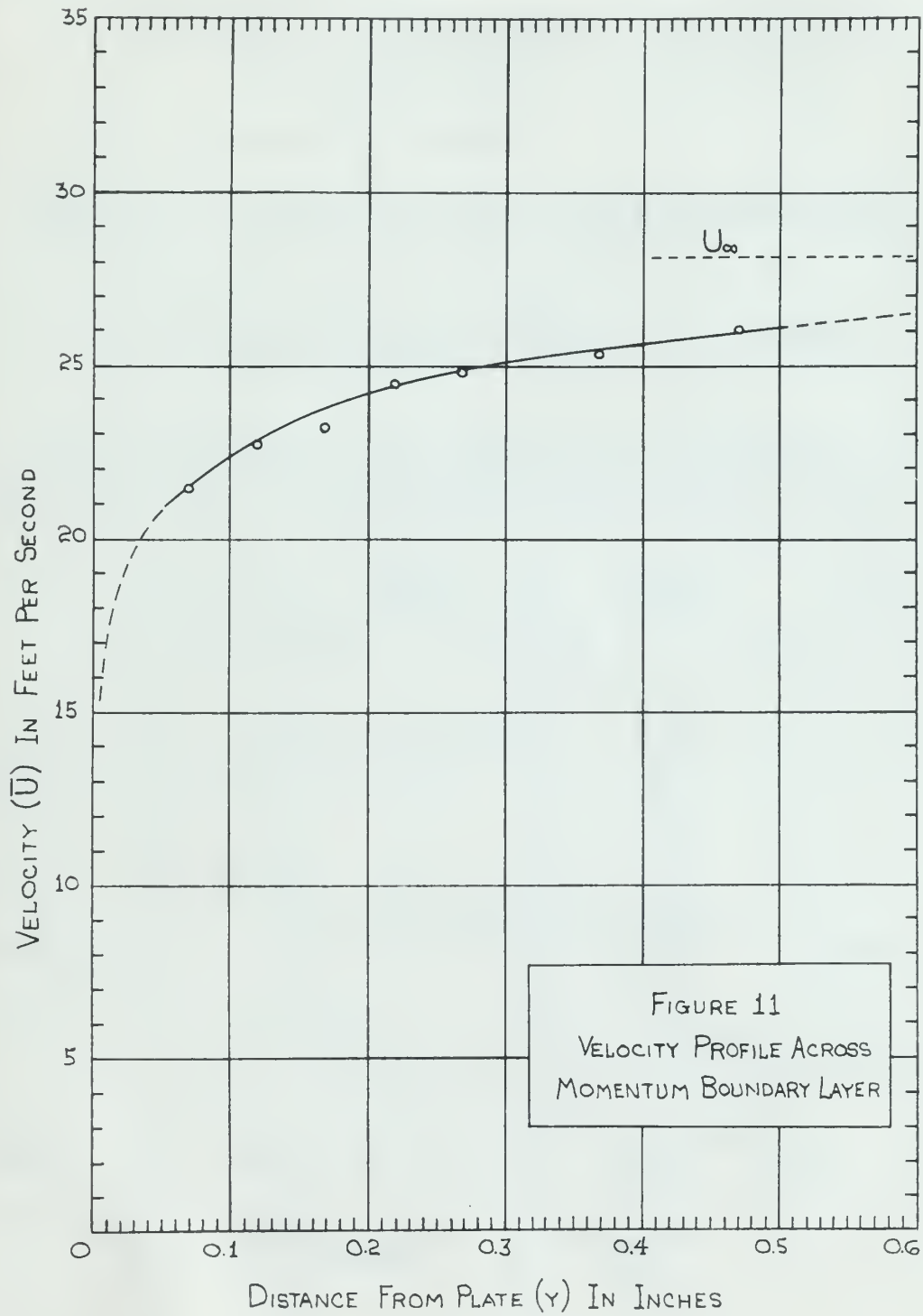
y (in)	x (in)	$\sqrt{\frac{y}{x}}$ (1/2)(sec) ^{1/2}	$\bar{u}$ (ft/sec)	$\frac{\bar{u}}{U_{\infty}}$
0.000"	44.00	0.00	17.40	0.778
0.110"	44.32	0.78	17.40	0.778
0.100"	44.48	0.81	17.40	0.778
0.210"	44.88	0.92	17.40	0.778
0.240"	44.78	0.98	17.40	0.778
0.340"	44.90	1.00	17.40	0.778
0.400"	45.12	1.10	17.40	0.778
0.420"	45.20	1.11	17.40	0.778

7. Boundary Layer Thickness

The following equation can be used to locate the boundary layer thickness,

$$\delta = 0.37x \left(\frac{U_{\infty}}{\nu} \right)^{-1/2}$$

Considering a point exactly-half inch from the leading edge of the plate and a free stream velocity of exactly-half inch per second, the thickness will be,



$$V_{\infty} = (26 \text{ ft/sec})(3600 \text{ sec/hr}) = 9.36 \times 10^4 \text{ ft/hr.}$$

$$\rho = \frac{(0.02901)(p \text{ mm Hg})}{(t^{\circ}\text{C} + 273.2^{\circ})} = \frac{(0.02901)(767.05)}{(298.0)} = 0.0747 \text{ lbs/ft}^3$$

$$\nu = \frac{\mu}{\rho} = \frac{0.460 \text{ lbs/ft-hr}}{0.0747 \text{ lbs/ft}^3} = 6.16 \frac{\text{ft}^2}{\text{hr}}$$

$$x = 29 \text{ inches} = 2.42 \text{ ft.}$$

$$\text{then } \delta_m = (0.37)(29 \text{ inches}) \left[\frac{(9.36 \times 10^4 \text{ ft/hr})(2.42 \text{ ft})}{(6.16 \text{ ft}^2/\text{hr})} \right]^{-1/5}$$

$$\delta_m = \frac{(10.73)}{(3.68 \times 10^4)^{1/5}} = \frac{(10.73)}{(8.13)} = 1.32 \text{ inches}$$

This thermal boundary layer thickness may be calculated in a similar manner employing the following equation taken from reference 14,

$$\delta_t = K_8 \left(\frac{V_{\infty} x}{\nu} \right)^{\frac{35b-36}{5(15-7b)}} \left(\frac{c_p \mu}{k} \right)^{\frac{7(c-1)}{15-7b}} (x)$$

$$\text{where, } K_8 = \left[\frac{0.37^{1/7} \times 72K}{7} \right]^{\frac{7}{15-7b}}$$

$$K = 0.0167; \quad b = 4/5; \quad c = 1/3$$

$$\text{substituting, } K_8 = \left[\frac{(0.37^{1/7})(72)(0.0167)}{7} \right]^{\frac{7}{15-5.6}} = 0.243$$

$$Pr = \frac{c_p \mu}{k} = 0.71 \quad \text{and } x = 10 \text{ inches}$$

$$\text{then, } \delta_t = 0.243 (3.68 \times 10^4)^{-0.1701} (0.71)^{-0.497} (10)$$

$$\delta_t = \frac{(2.43)}{(5.96)(0.844)} = 0.483 \text{ inches}$$

$$V_{90} = (20 \text{ ft/sec})(1000 \text{ sec/hr}) = 2.0 \times 10^4 \text{ ft/hr.}$$

$$\rho = \frac{(0.025)(0.01 \text{ ft})}{(0.01 + 0.01) \text{ ft}} = \frac{(0.00025)(0.01 \text{ ft})}{(0.02 \text{ ft})} = 0.000125 \text{ lb/ft}^3$$

$$\gamma = \frac{M}{\rho} = \frac{0.000125 \text{ lb/ft}^3}{0.01 \text{ ft}} = 0.0125 \frac{\text{lb}}{\text{ft}^2}$$

$$x = 20 \text{ inches} = 2.0 \text{ ft.}$$

$$\text{then } \delta_m = (0.37)(20 \text{ inches}) \left[\frac{(0.025 \times 10^4 \text{ ft/hr})(0.01 \text{ ft})}{(1.12 \text{ ft/hr})} \right]^{-1/2}$$

$$\delta_m = \frac{(10.77)}{(0.01 \times 10^4)} \text{ ft} = \frac{10.77}{(0.12)} = 1.23 \text{ inches}$$

This thermal boundary layer thickness may be calculated in a similar

manner employing the following equation taken from reference 14,

$$\delta_t = \frac{K}{V} \left(\frac{h_0 x}{k} \right)^{1/2} \quad (1)$$

$$\text{where, } K = \left[\frac{0.37}{1} \times \frac{1}{1 - 0.7} \right]^{1/2}$$

$$K = 0.697, \quad h_0 = 10, \quad c = 10$$

$$\text{substituting, } \delta_t = \left[\frac{(0.37)(10)(0.01 \text{ ft})}{1} \right]^{1/2} = 0.243$$

$$\delta_t = \frac{K}{V} = 0.71 \text{ and } x = 10 \text{ inches}$$

$$\text{then, } \delta_t = 0.37(2.0 \text{ ft}) = 0.74 \text{ ft} = 8.88 \text{ inches} \quad (10)$$

$$\delta_t = \frac{(8.88 \text{ inches})}{(0.02 \text{ ft})} = 0.444 \text{ inches}$$

8. uv Correlation in "Cold" Momentum Boundary Layer

In the cold plate condition, equation 7 may be used to calculate the quantity $\overline{uv}$.

$$\overline{uv} = - \frac{1}{2(I')^2} \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right]$$

Data was taken at two resistance ratios and averaged. The values of (I') were taken from section (b) of this appendix, while i_s was calculated for a bridge null setting of 483 by the method of section 4. The remaining data is shown in tabular form together with the results obtained, which are plotted in the RESULTS.

$\gamma = 1.2$						averaged values	
y (in)	$(M_{sn})_a$ (mv)	$(M_{scn})_a$ (mv)	(M_n) (mv)	$(M_{sn})_b$ (mv)	$(M_{scn})_b$ (mv)	$\overline{uv}$ (ft/sec) ²	$\frac{\overline{uv}}{U_\infty^2}$
0.068	24.2	61.2	7.50	31.0	62.8	-2.75	-0.00407
0.118	23.4	61.3	"	29.2	63.3	-2.48	-0.00367
0.168	21.8	61.2	"	27.3	62.6	-2.25	-0.00333
0.218	20.8	60.7	"	25.7	61.0	-1.82	-0.00269
0.268	19.4	60.2	"	24.8	61.5	-1.40	-0.00207
0.368	19.1	59.7	"	23.3	62.4	-1.29	-0.00191
0.468	17.0	59.0	"	19.6	61.8	-0.929	-0.00137

$\gamma = 1.4$					
0.068	60.9	128.0	7.45	79.5	137.0
0.118	59.0	128.5	"	73.4	126.5
0.168	53.5	125.6	"	72.5	131.0
0.218	50.3	124.0	"	66.3	129.5
0.268	47.8	123.0	"	60.2	130.5
0.368	45.5	122.5	"	60.3	127.0
0.468	40.2	122.3	"	54.5	124.0

3. The calculation of the average resistance ratio

In the case of the calculation, equation 7 may be used to calculate the

average ratio

$$\bar{R} = \frac{1}{n} \sum_{i=1}^n \left(\frac{R_i}{R_{ref}} \right) = \frac{1}{n} \left(\frac{R_1}{R_{ref}} + \frac{R_2}{R_{ref}} + \dots + \frac{R_n}{R_{ref}} \right)$$

Data was taken at two resistance ratios and averaged. The values of (1') were taken from section (b) of this appendix, while 1 was calculated for a bridge built using of ASD by the method of section A. The resulting data is shown in similar form together with the results obtained, which are plotted in the results.

$\gamma = 1.3$							
$\bar{R}$ (in)	$\bar{R}$ (in)	$\bar{R}$ (in)	$\bar{R}$ (in)	$\bar{R}$ (in)	$\bar{R}$ (in)	$\bar{R}$ (in)	$\bar{R}$ (in)
0.002	26.2	21.2	17.2	13.2	9.2	5.2	1.2
0.110	27.4	21.2	17.2	13.2	9.2	5.2	1.2
0.168	21.8	21.2	17.2	13.2	9.2	5.2	1.2
0.218	20.8	20.7	17.2	13.2	9.2	5.2	1.2
0.268	18.4	20.2	17.2	13.2	9.2	5.2	1.2
0.268	19.1	20.7	17.2	13.2	9.2	5.2	1.2
0.468	17.8	20.0	17.2	13.2	9.2	5.2	1.2

$\gamma = 1.4$

0.002	20.9	120.0	7.42	134.6
0.110	22.0	120.2	"	126.2
0.168	23.2	122.8	"	131.0
0.218	20.2	124.0	"	122.2
0.268	21.8	123.0	"	130.2
0.368	22.2	122.2	"	127.0
0.468	20.2	122.2	"	124.0

9. $\overline{uv}$ and $\overline{vt}$ Correlations in Thermal Boundary Layer

On the hot plate condition, equation 72 may be used to calculate the quantities $\overline{uv}$ and $\overline{vt}$. Rearranged,

$$\overline{uv} = \frac{\sqrt{2} (I_a + I_b)}{4 I'} \left[\frac{\alpha_e}{\gamma - [1 + \alpha_e (T_g - T_e)]} \right] = \frac{1}{2(I')^2} \left[\left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_b - \left(\frac{M_{sn}^2 - M_n^2}{M_{scn}^2 - M_{sn}^2} \right)_a \right]$$

Data was taken at two resistance ratios as for the previous calculations. Great difficulty was experienced in obtaining reasonable results because of the scattering of the data. To reduce the random errors present the data was plotted in figure 12 through 14, and faired into smooth curves. The remainder of the calculations were carried out with the smoothed data. The raw data, the faired data and the final results are tabulated below. Normalized curves of the results appear in figures 1 and 3.

Raw Data							
$\gamma = 1.2$							
y (in.)	$4I_a$ (ma)	$4I_b$ (ma)	M_n (mv)	$(M_{sn})_a$ (mv)	$(M_{sn})_b$ (mv)	$(M_{scn})_a$ (mv)	$(M_{scn})_b$ (mv)
0.068	121.0	127.8	7.5	41.7	60.1	61.7	76.5
0.118	121.1	129.9	"	41.2	50.2	61.0	70.2
0.168	125.0	132.2	"	34.3	39.9	58.7	62.6
0.218	126.1	132.3	"	28.1	33.5	56.5	53.7
0.268	128.0	131.8	"	25.1	29.5	53.7	57.5
0.368	128.8	132.9	"	19.7	24.4	52.2	53.5
0.468	130.1	132.8	"	17.7	21.9	52.2	53.0

2. uv and its Correlation to Thermal Boundary Layer

On the hot plate condition, equation 2 may be used to calculate

the quantities uv and w . Rearranged,

$$uv = \frac{\sqrt{2} (I_u + I_p)}{2 I_p} \left[\frac{1}{1 + \frac{1}{2} \left(\frac{I_u + I_p}{I_p} \right)^2} \right] = \frac{\frac{1}{2} \left(\frac{I_u + I_p}{I_p} \right)^2}{1 + \frac{1}{2} \left(\frac{I_u + I_p}{I_p} \right)^2} \left(\frac{I_u + I_p}{I_p} \right) = \frac{\frac{1}{2} \left(\frac{I_u + I_p}{I_p} \right)^2}{1 + \frac{1}{2} \left(\frac{I_u + I_p}{I_p} \right)^2} \left(\frac{I_u + I_p}{I_p} \right)$$

Data was taken at two resistance ratios as for the previous calculations.

Great difficulty was experienced in obtaining reproducible results because of

the wandering of the data. To reduce the random errors present the data

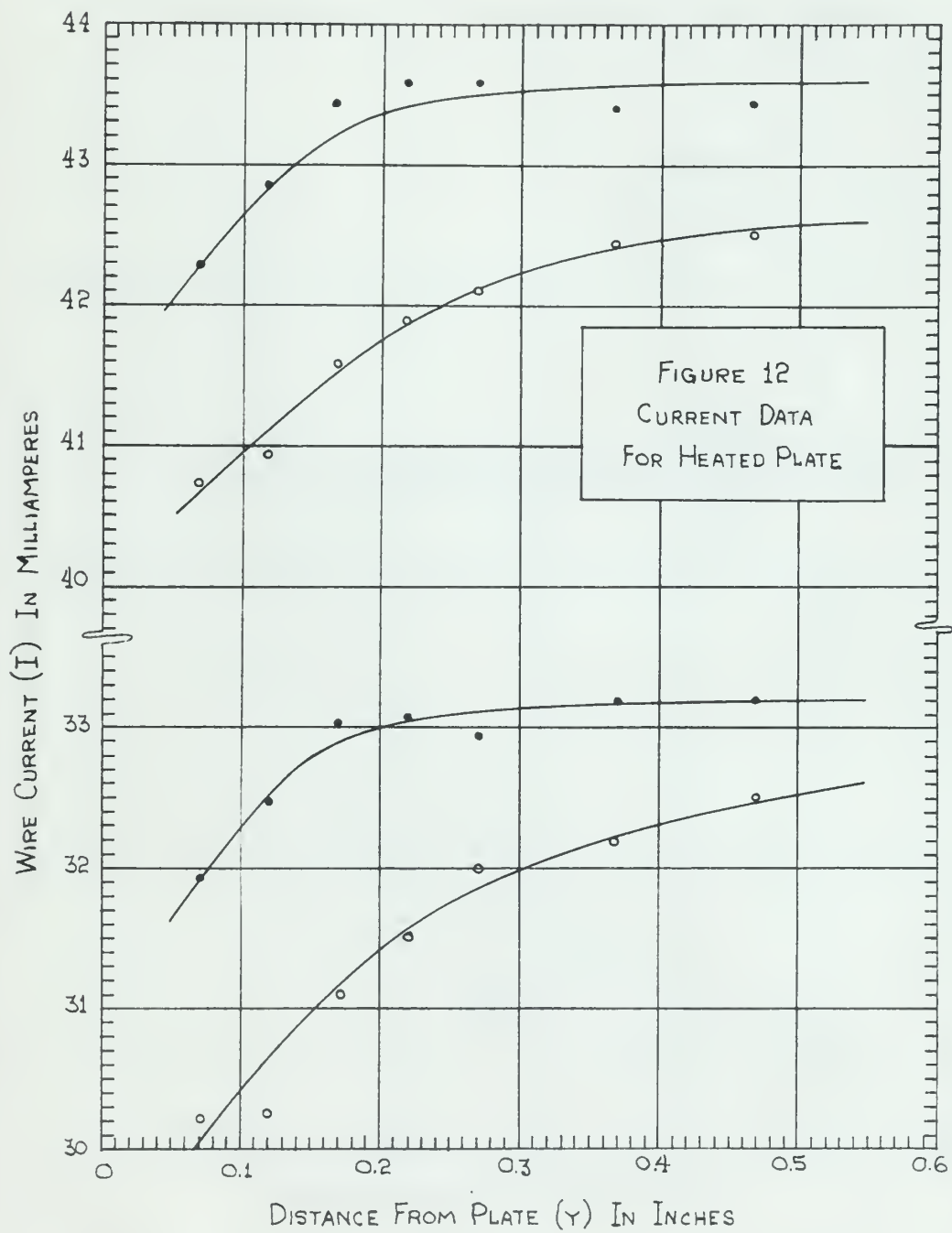
was plotted in Figure 12 through 14, and fitted into smooth curves. The

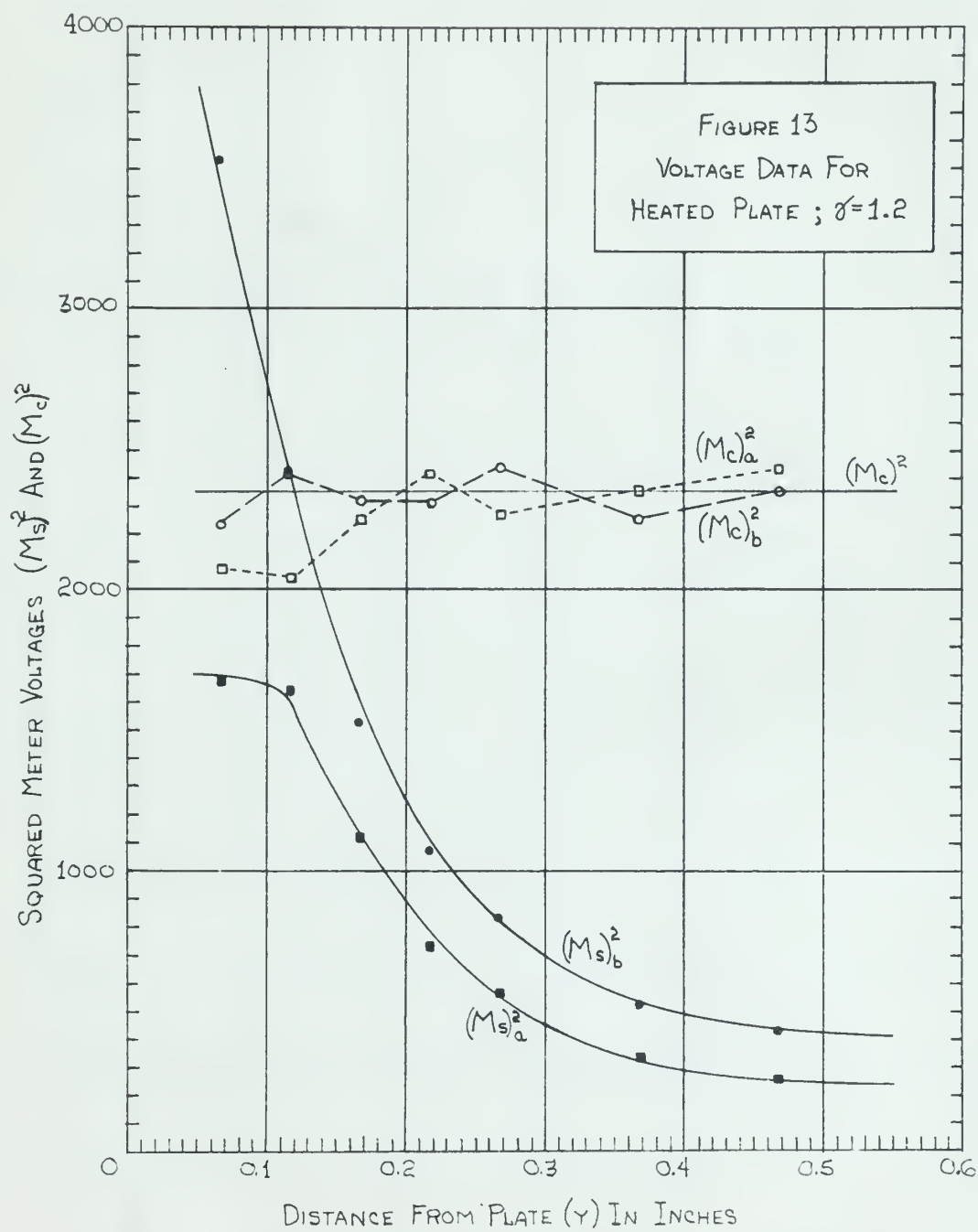
remainders of the calculations were plotted and with the smoothed data. The

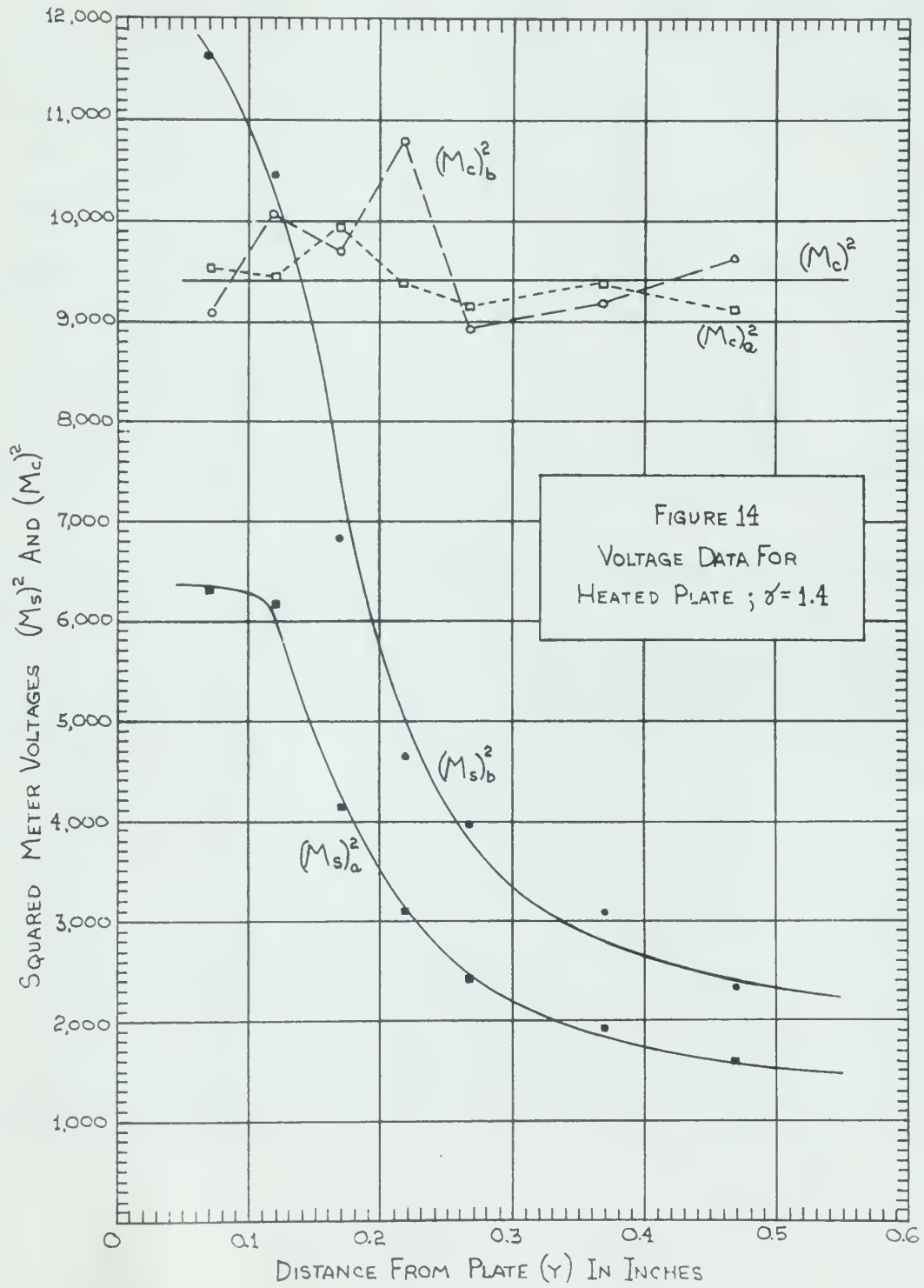
raw data, the fitted data and the final results are tabulated below. Normal-

ized curves of the results appear in Figures 1 and 2.

Raw Data							
$\gamma = 1.2$							
γ (%)	I_p (w)	I_u (w)	I_{up} (w)	I_{up} (w)	I_{up} (w)	I_{up} (w)	I_{up} (w)
0.062	121.0	127.8	7.2	41.7	69.1	67.7	70.2
0.110	121.1	127.9	"	41.2	70.2	67.0	70.2
0.168	122.0	128.2	"	36.3	70.9	70.7	63.6
0.218	126.1	128.3	"	28.1	72.2	72.2	53.7
0.266	126.0	131.8	"	22.1	74.2	72.7	57.2
0.328	128.2	132.3	"	19.7	74.2	71.2	53.2
0.461	130.1	132.8	"	17.7	71.8	72.2	53.0







Raw Data, con.

$$\gamma = 1.4$$

y (in)	$4I_a$ (ma)	$4I_b$ (ma)	M_n (mv)	$(M_{sn})_a$ (mv)	$(M_{sn})_b$ (mv)	$(M_{scn})_a$ (mv)	$(M_{scn})_b$ (mv)
0.068	163.0	169.2	7.5	79.7	108.0	126.0	144.0
0.118	163.8	171.4	"	78.5	102.5	125.0	143.5
0.168	166.3	173.3	"	64.7	83.1	119.0	129.0
0.218	167.7	174.3	"	56.0	68.6	112.0	124.5
0.268	168.5	174.3	"	49.8	63.6	108.0	114.0
0.368	169.8	173.6	"	44.5	56.0	106.5	111.0
0.468	170.0	173.7	"	41.0	49.3	104.0	110.0

Fairred Data

$$\gamma = 1.2$$

y (in)	I_a (ma)	I_b (ma)	$(M_s)_a^2$ (mv) ²	$(M_s)_b^2$ (mv) ²	$(M_c)^2$ (mv) ²
0.068	30.01	31.93	1690	3520	2350
0.118	30.65	32.51	1620	2410	"
0.168	31.18	32.89	1130	1600	"
0.218	31.60	33.06	770	1100	"
0.268	31.88	33.13	530	810	"
0.368	32.23	33.19	330	550	"
0.468	32.47	33.20	240	440	"

Raw Data, cont.

 $\gamma = 1.4$

V (mV)	I_B (mA)	I_D (mA)	M_{D1} (mV)	M_{D2} (mV)	M_{D3} (mV)	M_{D4} (mV)
0.068	100.0	100.0	7.2	78.7	108.0	144.0
0.118	100.0	100.0	"	78.7	108.0	144.0
0.168	100.0	100.0	"	78.7	108.0	144.0
0.218	100.0	100.0	"	78.7	108.0	144.0
0.268	100.0	100.0	"	78.7	108.0	144.0
0.368	100.0	100.0	"	78.7	108.0	144.0
0.468	100.0	100.0	"	78.7	108.0	144.0

Raw Data

 $\gamma = 1.5$

V (mV)	I_B (mA)	I_D (mA)	M_{D1} (mV)	M_{D2} (mV)	M_{D3} (mV)
0.068	30.01	31.83	1000	3250	3320
0.118	30.02	32.71	1000	3250	3320
0.168	31.10	32.88	1130	3250	3320
0.218	31.60	33.06	1200	3250	3320
0.268	31.88	33.13	1230	3250	3320
0.368	32.23	33.19	1300	3250	3320
0.468	32.47	33.20	1340	3250	3320

Paired Data, con.

$$\gamma = 1.4$$

y (in)	I_a (ma)	I_b (ma)	$(M_s)_a^2$ (mv) ²	$(M_s)_b^2$ (mv) ²	$(M_c)^2$ (mv) ²
0.068	40.87	42.30	6320	11650	9400
0.118	41.13	42.81	6040	10250	"
0.168	41.55	43.22	4190	7380	"
0.218	41.89	43.42	3130	5000	"
0.268	42.10	43.50	2470	3840	"
0.368	42.41	43.56	1830	2790	"
0.468	42.56	43.58	1600	2400	"

Results

y (in)	$\overline{uv}$ (ft/sec) ²	$\overline{\frac{uv}{U_\infty^2}}$	$\overline{vt}$ (ft °C/sec)	$\overline{v_\infty(T_s - T_e)}$
0.068	+4.69	+0.00694	+7.96	+0.0383
0.118	-2.74	- 0.00405	+1.74	+0.00837
0.168	-3.35	-0.00496	+0.421	+0.00202
0.218	-1.41	-0.00209	+0.633	+0.00304
0.268	-0.641	-0.00095	+0.737	+0.00354
0.368	-0.205	-0.00030	+0.686	+0.00330
0.468	+0.002	+0.00000	+0.693	+0.00333

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λ (m)	I_a (mV)	I_p (mV)	(M_s) (mV)	(M_d) (mV)	(M_c) (mV)
0.008	40.87	43.30	0.030	11.020	0.000
0.118	41.13	43.81	0.040	10.320	"
0.168	41.22	43.33	0.030	9.380	"
0.218	41.52	43.43	0.030	8.000	"
0.268	43.10	43.20	0.040	3.480	"
0.368	43.41	43.26	0.030	3.280	"
0.468	43.26	43.28	0.000	2.400	"

Results

λ (m)	$(\frac{I_a}{I_p})$ (sec/sec)	$(\frac{M_s}{M_d})$ (sec/sec)	$(\frac{M_c}{M_d})$ (sec/sec)
0.008	+4.02	+0.0034	+1.26
0.118	-5.24	-0.00402	+1.24
0.168	-3.32	-0.00488	+0.431
0.218	-1.41	-0.00303	+0.633
0.268	-0.941	-0.00032	+0.337
0.368	-0.302	-0.00030	+0.686
0.468	+0.002	+0.00000	+0.633

APPENDIX F

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